

Center for  
Electronic Correlations and Magnetism  
University of Augsburg

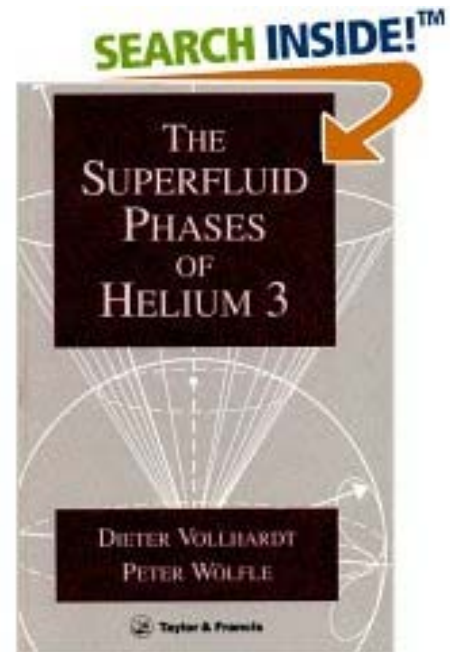
# Superfluid Helium-3: From very low Temperatures to the Big Bang

Dieter Vollhardt

Group Seminar, Theoretical Physics III  
November 4, 2009

## Contents:

- The quantum liquids  $^3\text{He}$  and  $^4\text{He}$
- Superfluid phases of  $^3\text{He}$
- Broken symmetries and long-range order
- Topologically stable defects
- Big Bang simulation in the low temperature lab



*The Superfluid Phases of Helium 3*

D. Vollhardt and P. Wölfle

(Taylor & Francis, 1990)

# Helium

Two stable Helium isotopes:

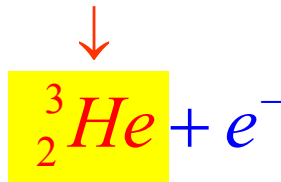
${}^4\text{He}$ : air, oil wells, ...

Janssen/Lockyer (1868)

Ramsay (1895)



(1939)



$$\left. \frac{\text{He}}{\text{air}} \approx 5 \times 10^{-6}, \quad \frac{{}^3\text{He}}{{}^4\text{He}} \right|_{\text{air}} \approx 1 \times 10^{-6}$$

Research on macroscopic samples of  ${}^3\text{He}$  since 1947

# Helium

Atoms: spherical, hard core diameter  $\sim 2.5 \text{ \AA}$

Interaction: 

- hard sphere **repulsion**
- van der Waals dipole/multipole **attraction**

Boiling point: 4.2 K,  $^4\text{He}$  Kamerlingh Onnes (1908)

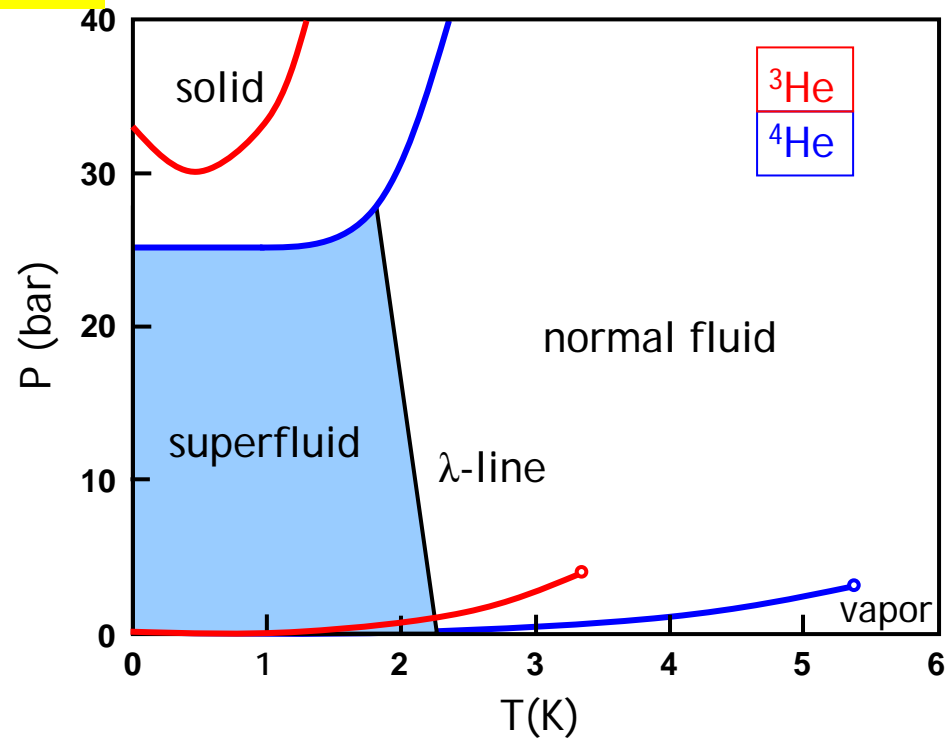


Nobel Prize 1913

3.2 K,  $^3\text{He}$  Sydoriak, *et al.* (1949)

Dense, simple liquid { isotropic  
short-range interactions  
extremely pure

# Helium

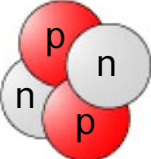
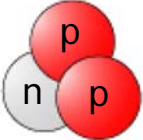


- Atoms:
- spherical shape → weak attraction
  - light mass → strong zero-point motion

$T \rightarrow 0, P \lesssim 30$  bar: Helium remains liquid

$$\lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \rightarrow 0} \text{Macroscopic quantum phenomena}$$

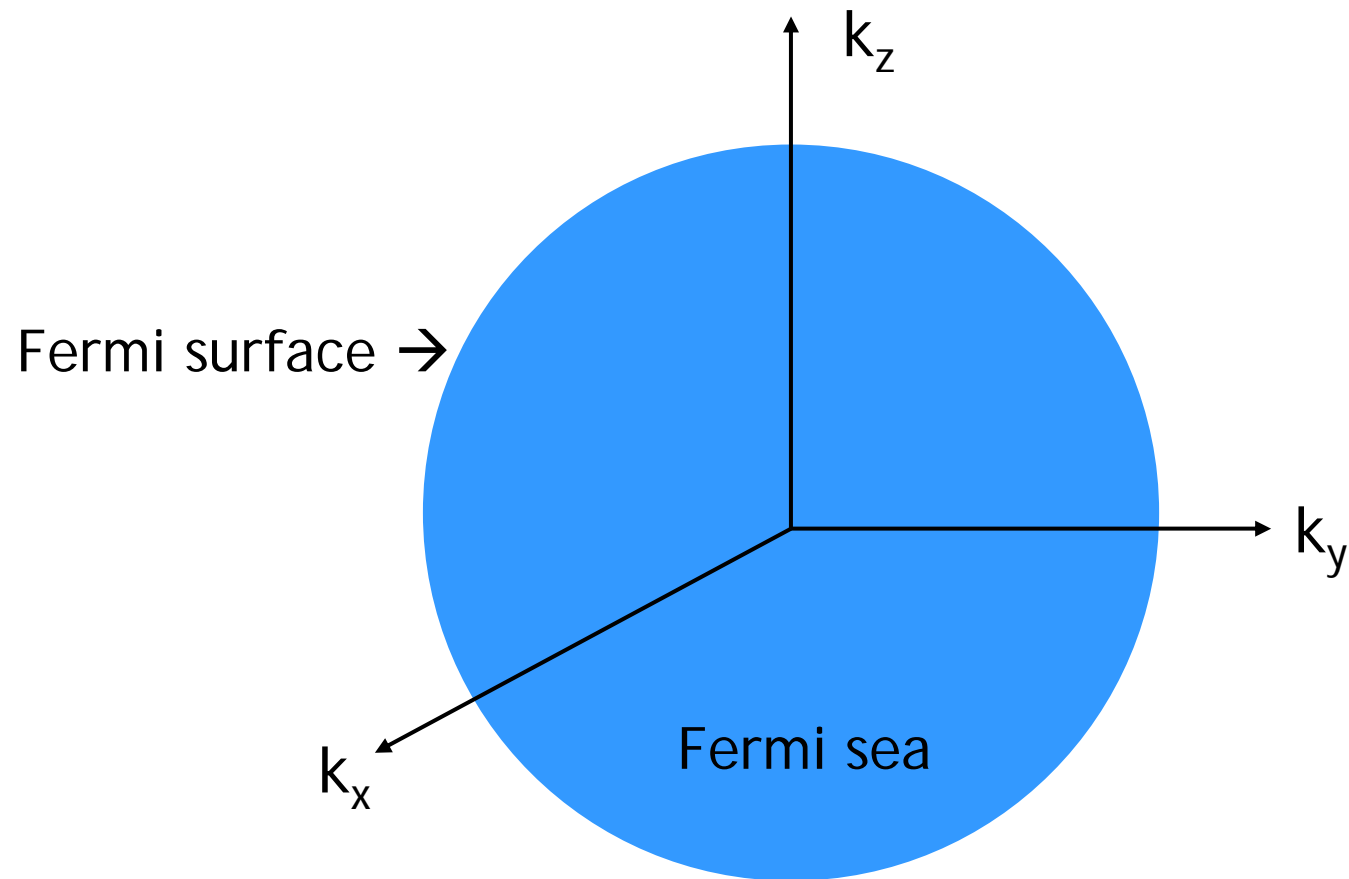
# Helium

	$^4\text{He}$	$^3\text{He}$
Electron shell:	$2 e^-$ , $S = 0$	
Nucleus:	 $S = 0$	 $S = \frac{1}{2}\hbar$
Atom(!) is a	Boson	Fermion
Phase transition	$T_\lambda = 2.2 \text{ K}$ ("BEC")	$T_c = ???$

Quantum liquids

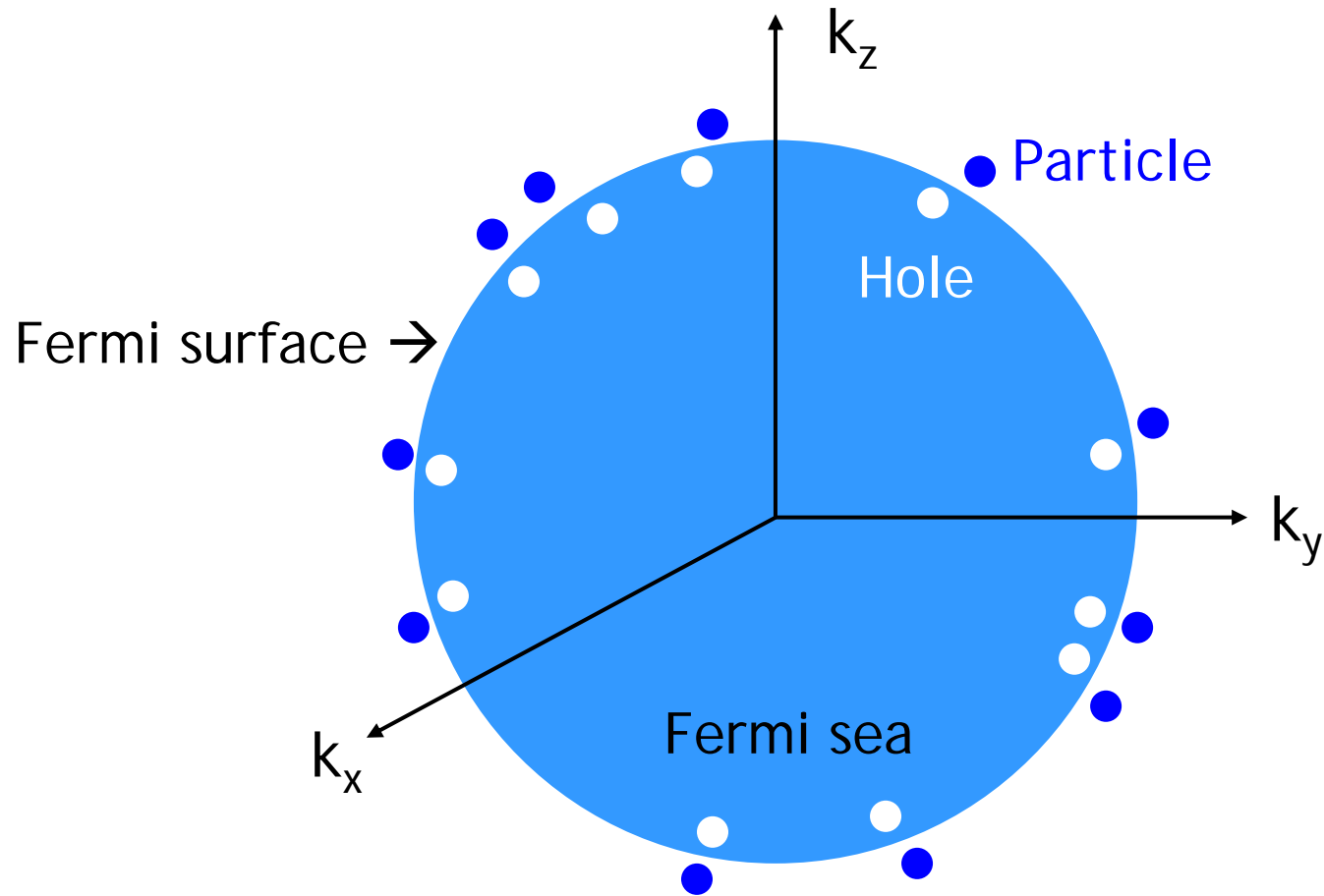
Fermi liquid theory

# Fermi gas: Ground state





Fermi gas: Excited states ( $T > 0$ )

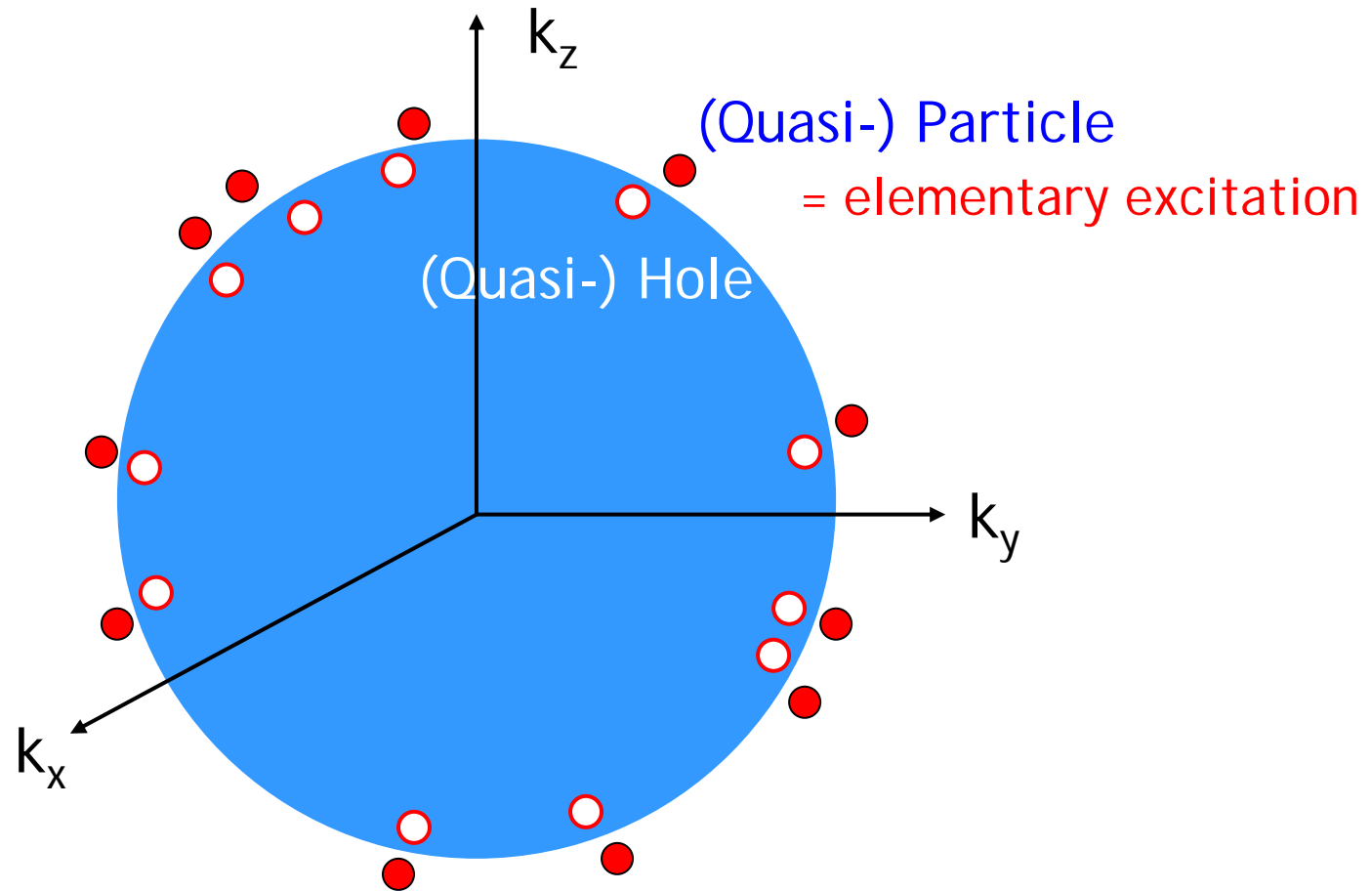


Exact  $k$ -states ("particles"): infinite life time

Switch on interaction adiabatically ( $d=3$ )

# Landau Fermi liquid

1-1 correspondence  
between k-states

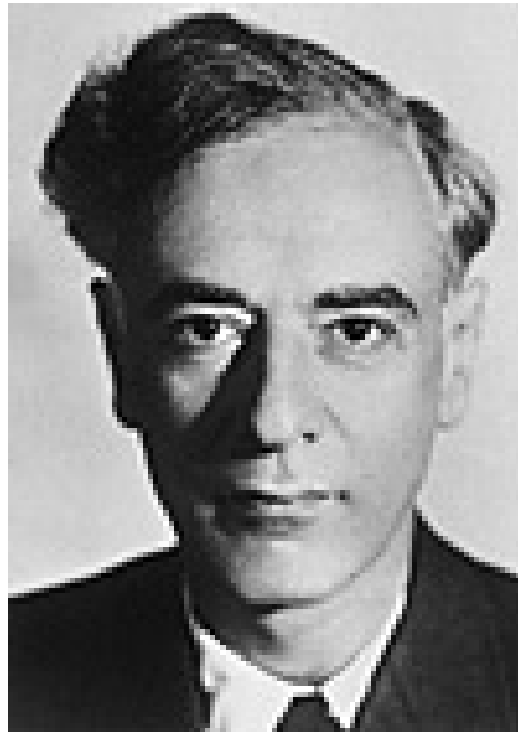


Prototype: Helium-3

- Large effective mass
- Strongly enhanced spin susceptibility
- Strongly reduced compressibility

## The Nobel Prize in Physics 1962

"for his pioneering theories for condensed matter,  
especially liquid helium"

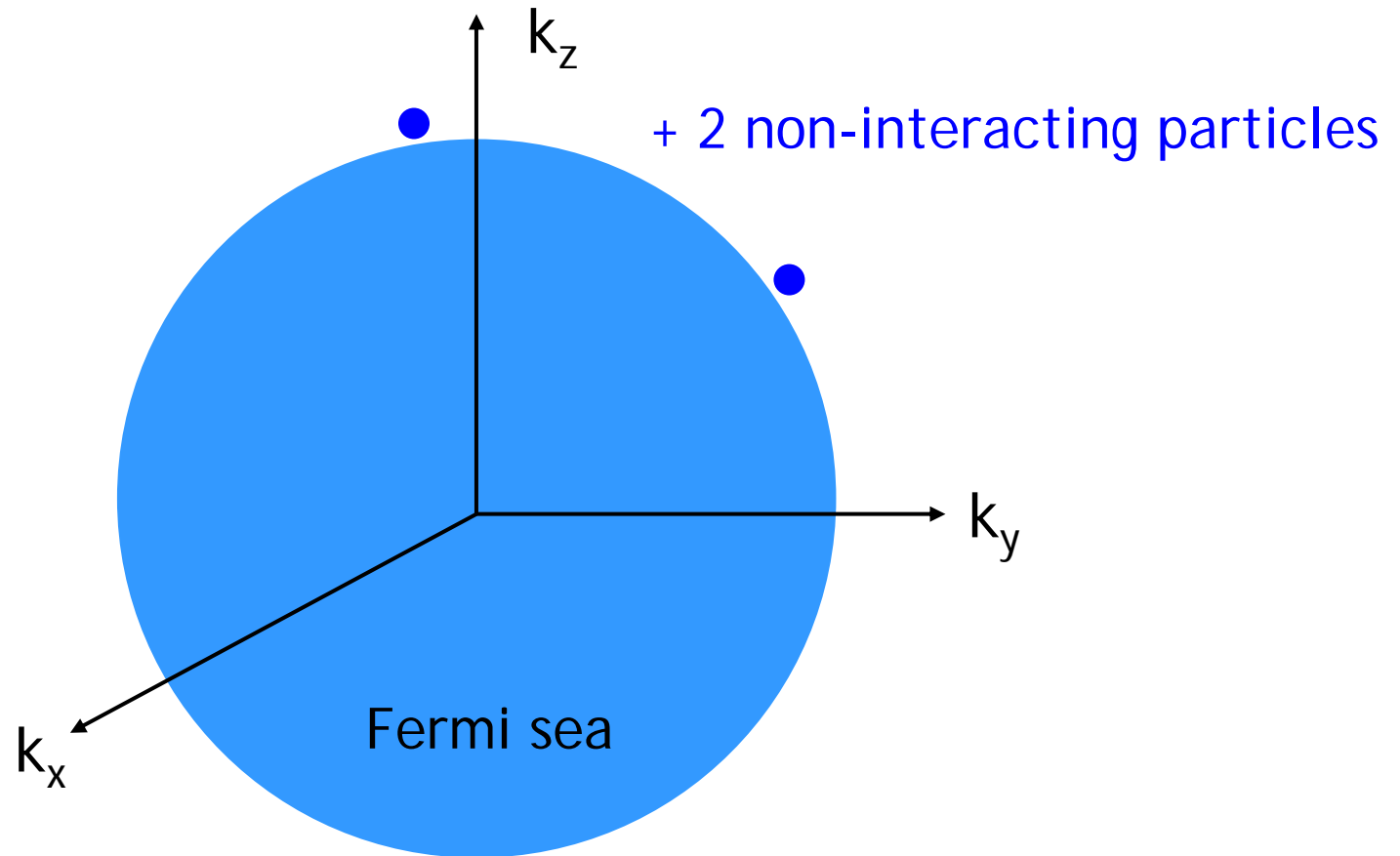


Lev Davidovich Landau

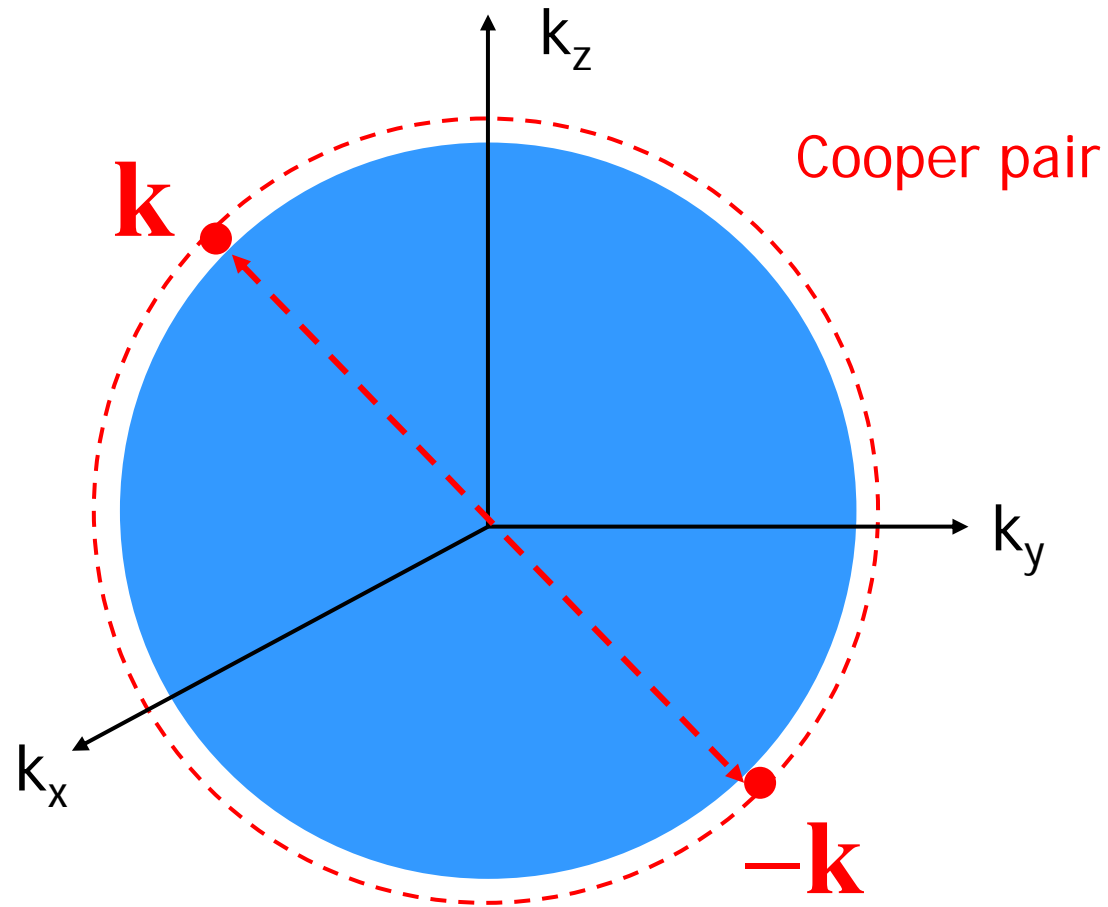
USSR

b. 1908, d. 1968

# Instability of Landau Fermi liquid

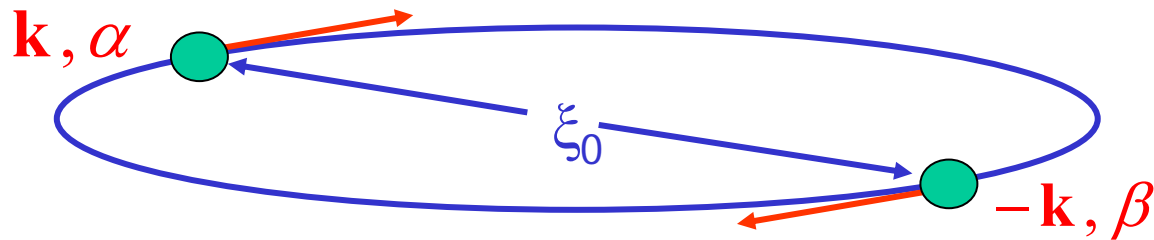


Arbitrarily weak attraction  $\Rightarrow$  Cooper instability



Universal fermionic property

Arbitrarily weak attraction  $\Rightarrow$  Cooper pair  $(\mathbf{k}, \alpha; -\mathbf{k}, \beta)$



$$\Psi_{L=0,2,4,\dots} = \psi(\mathbf{r}) |\uparrow\downarrow - \downarrow\uparrow\rangle$$

$S=0$  (singlet)

$$\begin{aligned} \Psi_{L=1,3,5,\dots} = & \psi_+(\mathbf{r}) |\uparrow\uparrow\rangle \\ & + \psi_0(\mathbf{r}) |\uparrow\downarrow + \downarrow\uparrow\rangle \\ & + \psi_-(\mathbf{r}) |\downarrow\downarrow\rangle \end{aligned}$$

$S=1$  (triplet)

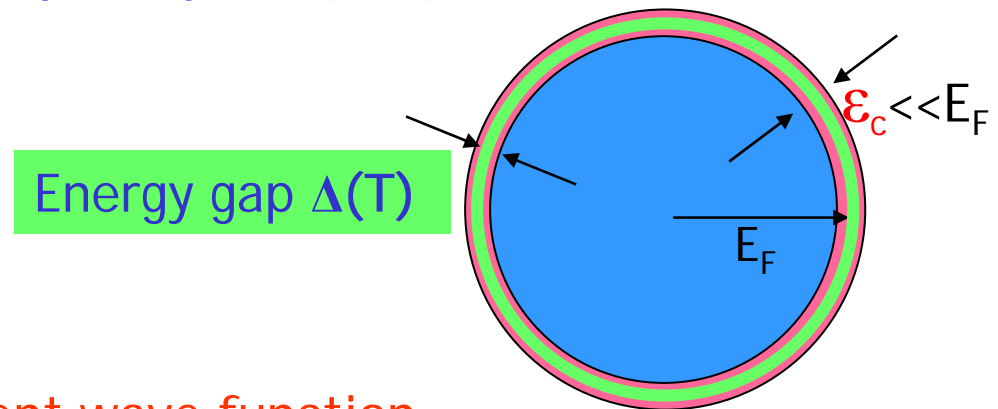
$L = 0$ : isotropic wave function  
 $L > 0$ : anisotropic wave function

Helium-3: Strongly repulsive interaction  $\rightarrow L > 0$  expected

# BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs



→ "Pair condensate"  
with macroscopically coherent wave function

Transition temperature

$$T_c = 1.13 \epsilon_c \exp(-1 / N(0) |V_L|)$$

"weak coupling theory"

$\epsilon_c, V_L$ : Magnitude ? Origin ? →  $T_c$  ?

Thanksgiving 1971: Transition in  $^3\text{He}$  at  $T_c = 0.0026 \text{ K}$

Osheroff, Richardson, Lee (1972)

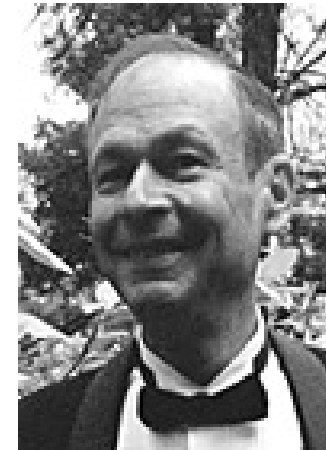
**The Nobel Prize in Physics 1996**  
"for their discovery of superfluidity in helium-3"



**David M. Lee**  
Cornell (USA)



**Douglas D. Osheroff**  
Stanford (USA)



**Robert C. Richardson**  
Cornell (USA)

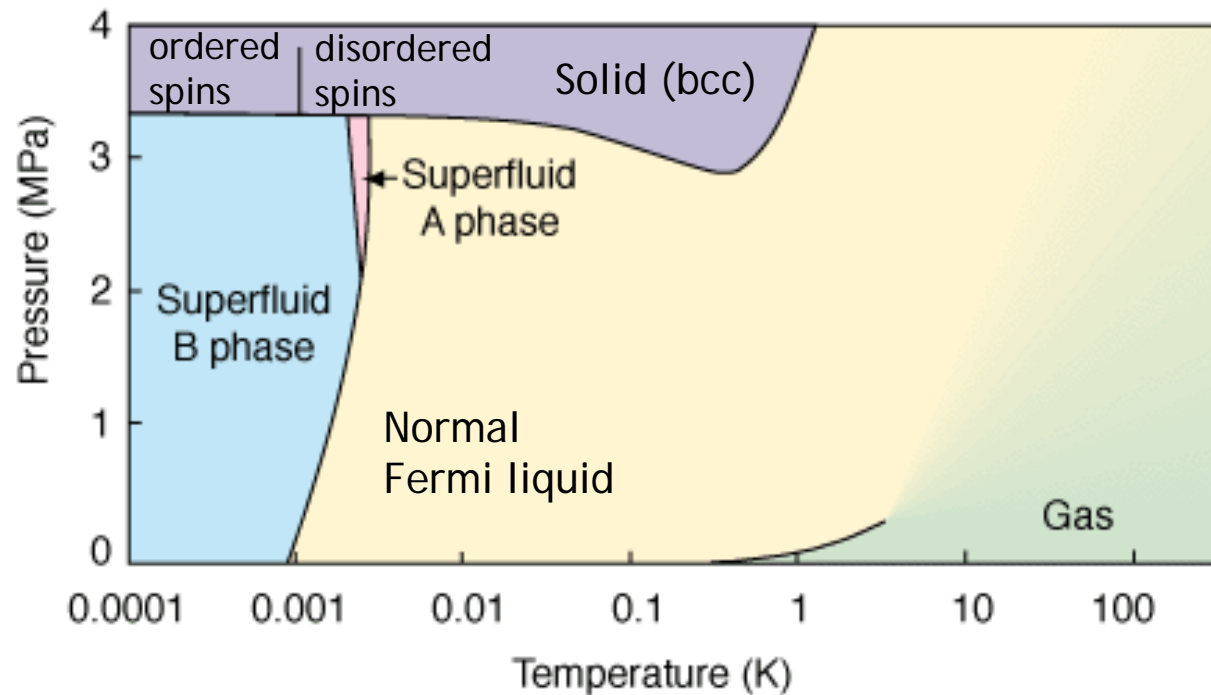


# Phase diagram of Helium-3

## P-T phase diagram

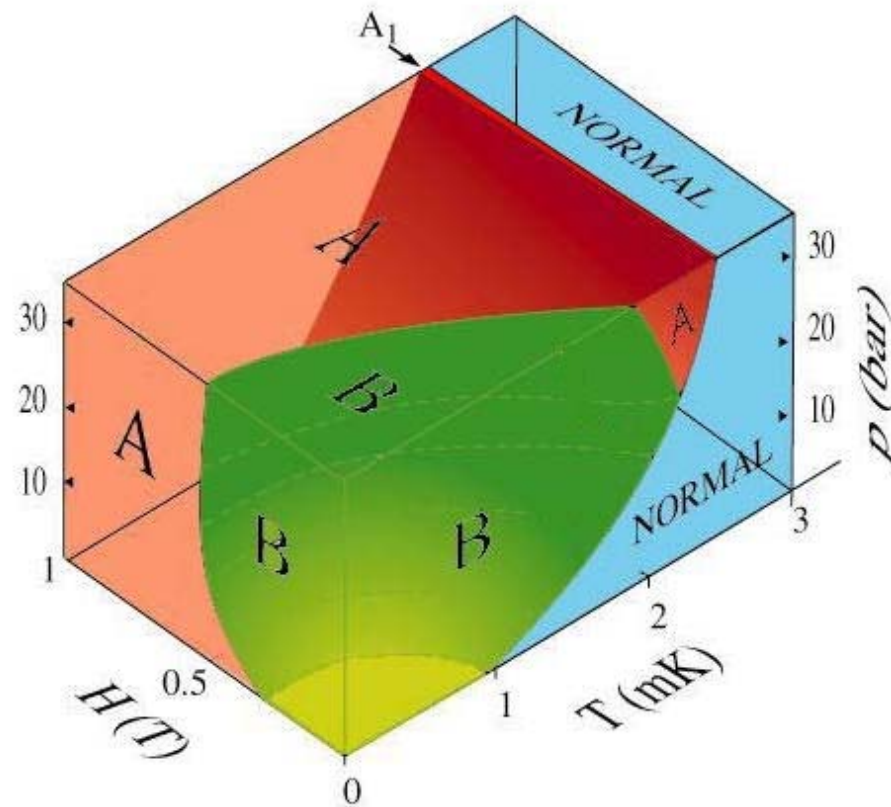
Dense, simple liquid

isotropic  
short-range interactions  
extremely pure  
nuclear spin  $S=1/2$



# Phase diagram of Helium-3

## P-T-H phase diagram



“Very low temperatures”:  $T \ll T_{\text{boiling}} \sim 3\text{-}4\text{ K}$   
 $\ll T_{\text{backgr. rad.}} \sim 3\text{ K}$

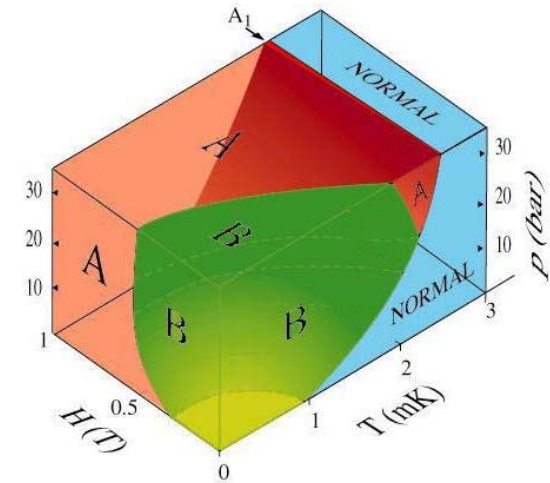
# Superfluid phases of $^3\text{He}$

Theory + experiment:  $L=1$ ,  $S=1$  in all phases

Leggett

Wölfle

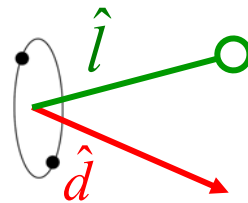
Mermin, ...



Attraction due to **spin fluctuations**

Anderson, Brinkman (1973)

→ anisotropy directions  
in a  $^3\text{He}$  Cooper pair



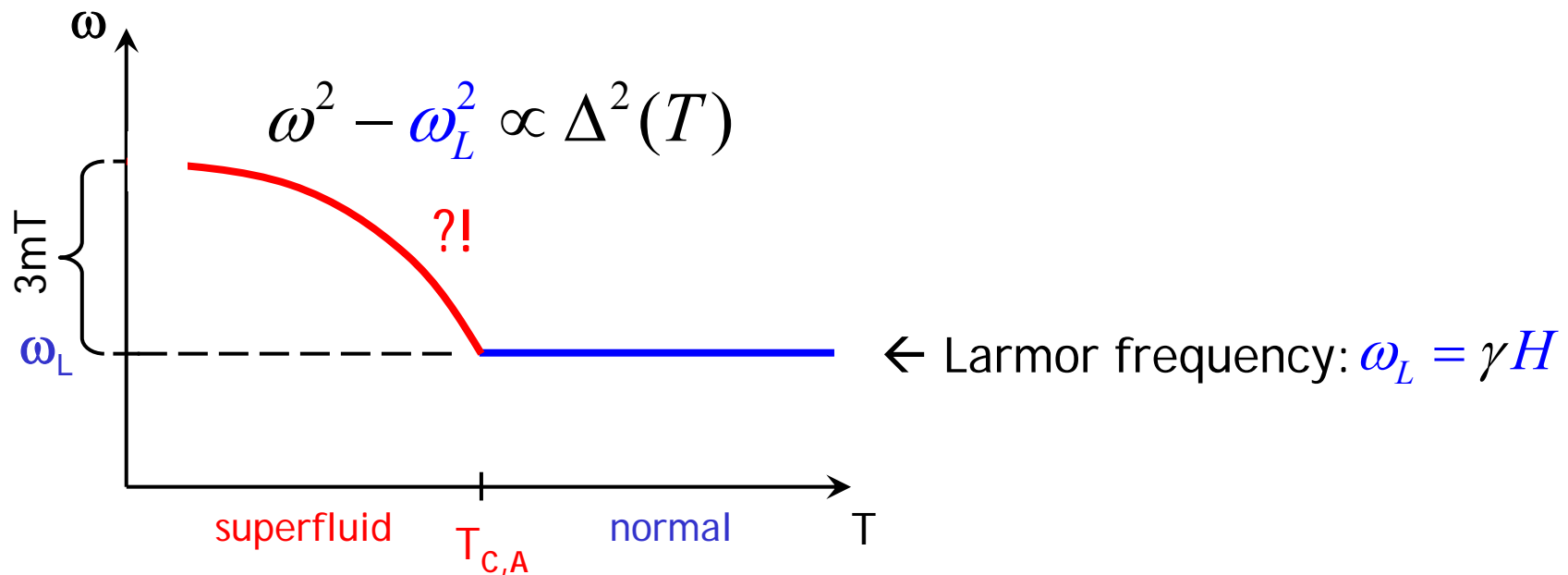
orbital part

spin part

... and a mystery!

NMR experiment on nuclear spins  $I = \frac{1}{2} \hbar$

Osheroff *et al.* (1972)

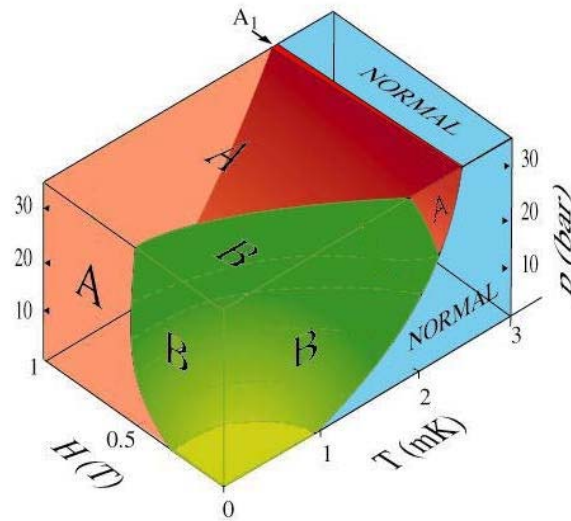


Shift of  $\omega_L$   $\Leftrightarrow$  spin-nonconserving interactions  
→ nuclear dipole interaction  $g_D \sim 10^{-7} K \ll T_C$

Origin of frequency shift ?!

Leggett (1973)

# The superfluid phases of $^3\text{He}$

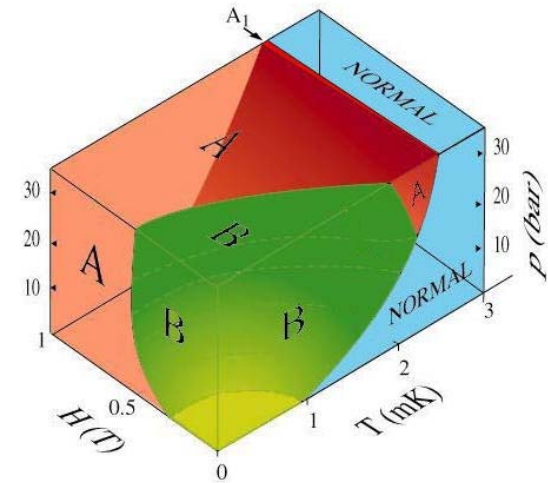
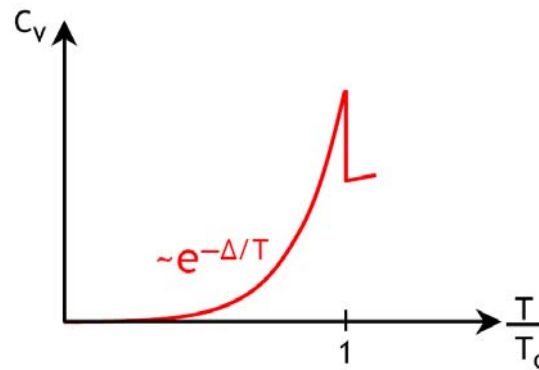
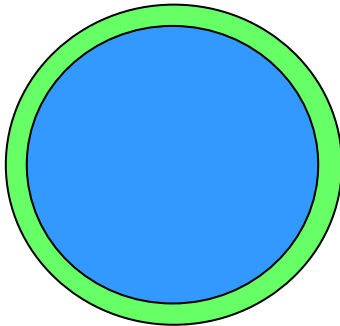


# B-phase

$$\Psi = |\uparrow\uparrow\rangle + |\uparrow\downarrow + \downarrow\uparrow\rangle + |\downarrow\downarrow\rangle$$

$$\Delta(\mathbf{k}) = \Delta_0$$

Balian, Werthamer (1963)  
Vdovin (1963)



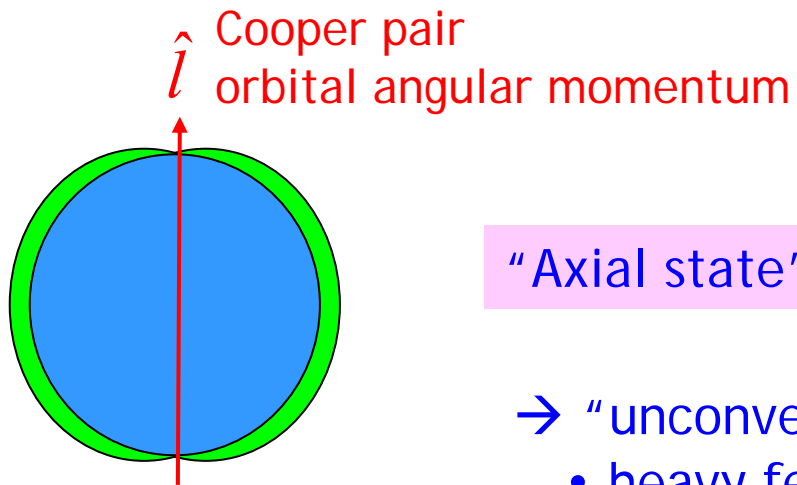
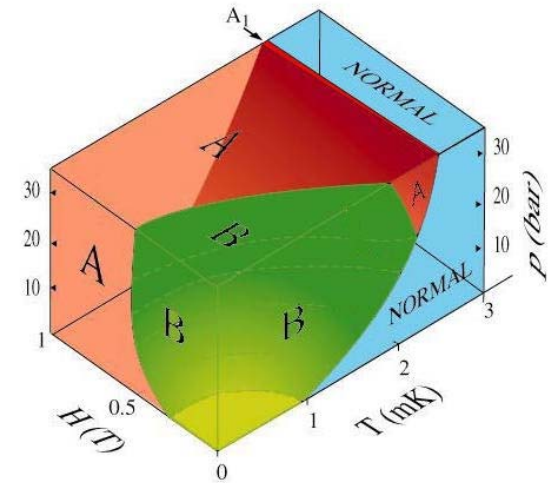
(pseudo-) isotropic state  $\leftrightarrow$  s-wave superconductor

Weak-coupling theory: stable for all  $T < T_c$

## A-phase

$$\Psi = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \rightarrow \text{strong anisotropy}$$

$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l}) \quad \text{Anderson, Morel (1961)}$$



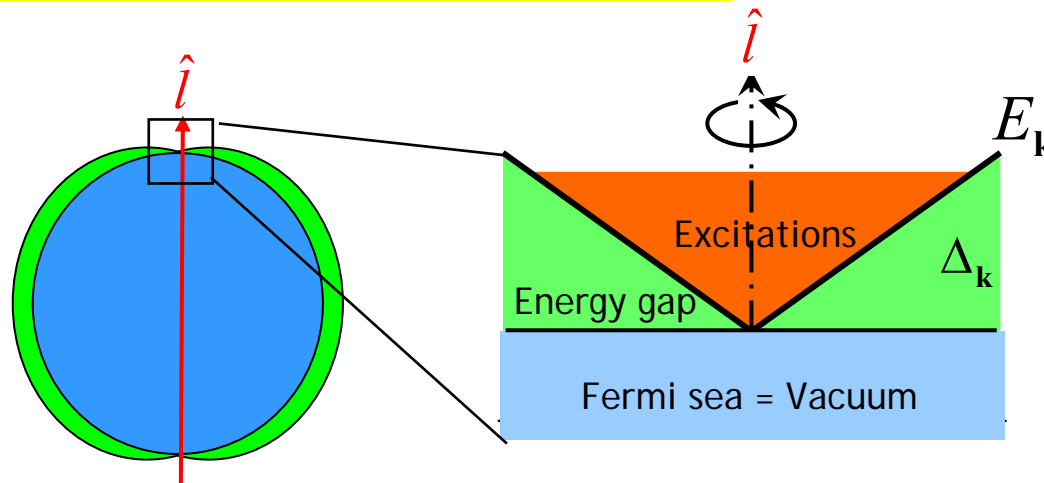
“Axial state” has point nodes

- “unconventional” pairing in
- heavy fermion/high- $T_c$  superconductors
  - $\text{Sr}_2\text{RuO}_4$

Strong-coupling effect

# <sup>3</sup>He-A: Spectrum near poles

Volovik (1987)



$$E_{\mathbf{k}}^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \end{cases} \quad \text{2 chiralities}$$

$$g^{ij} = v_F^2 l_i l_j + \left( \frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

$$\mathbf{A} = k_F \hat{\mathbf{l}}$$

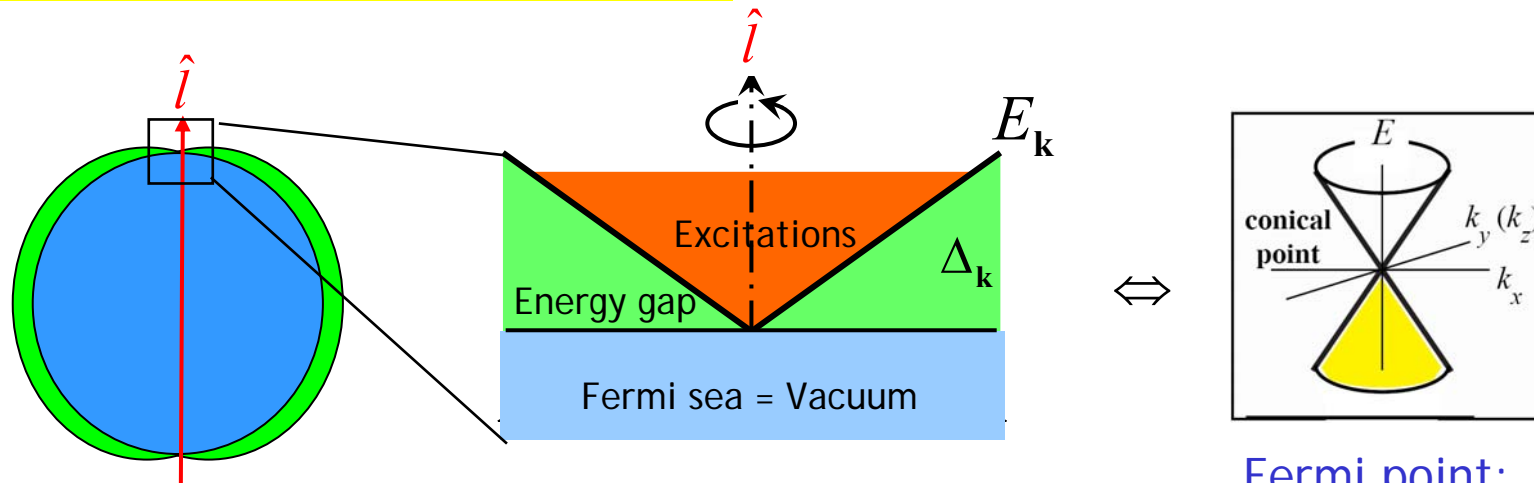
$$\mathbf{p} = \mathbf{k} - e\mathbf{A}$$

Lorentz invariance:  
Symmetry enhancement  
at low energies



# <sup>3</sup>He-A: Spectrum near poles

Volovik (1987)



Fermi point:  
spectral flow

$$E_{\mathbf{k}}^2 = v_F^2 (\mathbf{k} - \mathbf{k}_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \end{cases} \quad \text{2 chiralities}$$

$$g^{ij} = v_F^2 l_i l_j + \left( \frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

⇔ Massless, chiral leptons, e.g., neutrino  $E(\mathbf{p}) = cp$

→ Chiral anomaly of standard model

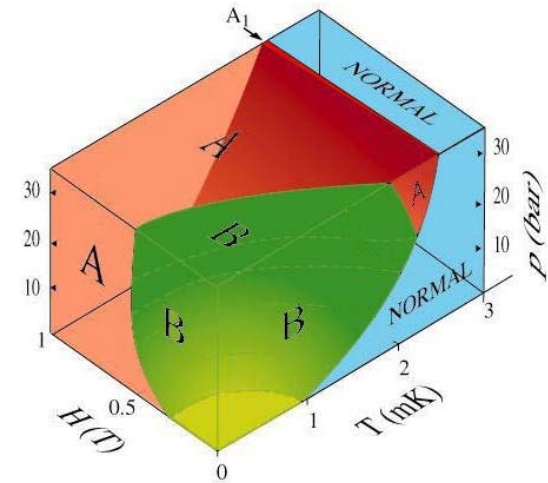
*The Universe in a Helium Droplet,*  
Volovik (2003)

$A_1$ -phase

finite magnetic field

$$\Psi = |\uparrow\uparrow\rangle$$

Long-range ordered magnetic liquid

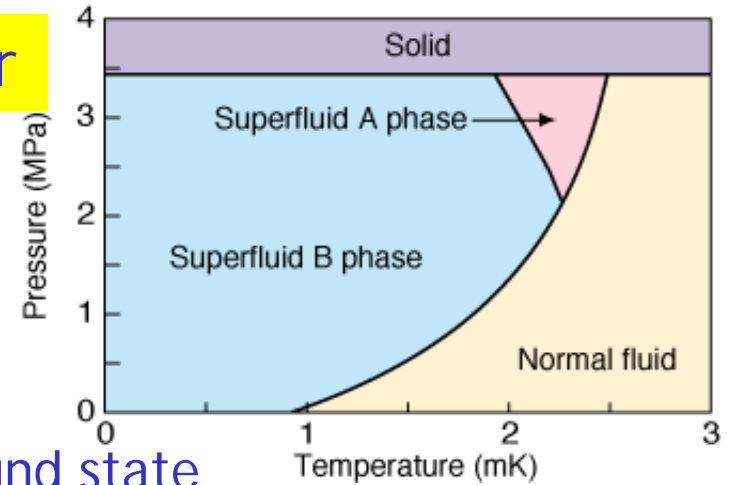


# Broken Symmetries, Long Range Order



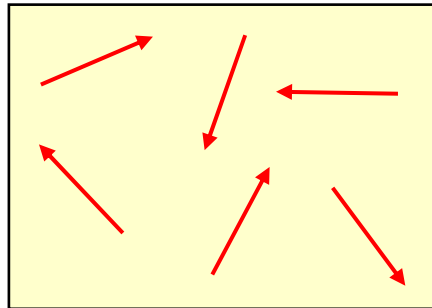
# Broken Symmetries, Long Range Order

Normal  $^3\text{He} \leftrightarrow ^3\text{He-A}, ^3\text{He-B}$ :  
 2. order phase transition



$T < T_c$ : higher order, lower symmetry of ground state

## I. Ferromagnet

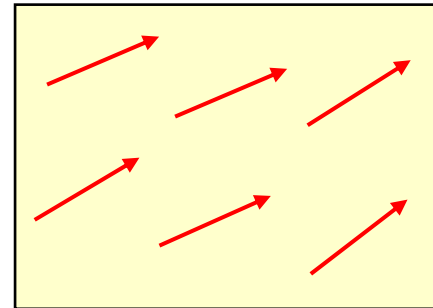


$T > T_c$

Average magnetization:  
 Symmetry group:

$$\langle \mathbf{M} \rangle = 0$$

$$SO(3)$$



$T < T_c$

$$\langle \mathbf{M} \rangle \neq 0 \quad \text{Order parameter}$$

$$U(1) \subset SO(3)$$

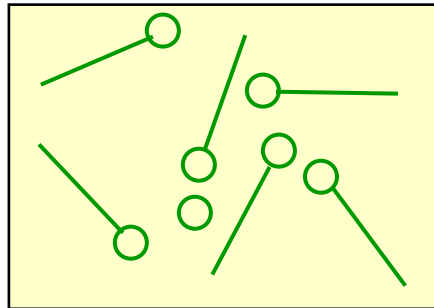
$T < T_c$ :  $SO(3)$  rotation symmetry in spin space spontaneously broken

# Broken Symmetries, Long Range Order

## 2. order phase transition

$T < T_c$ : higher order, lower symmetry of ground state

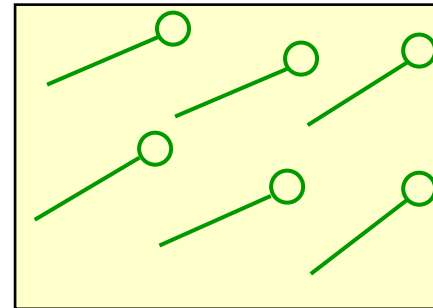
### II. Liquid crystal



$T > T_c$

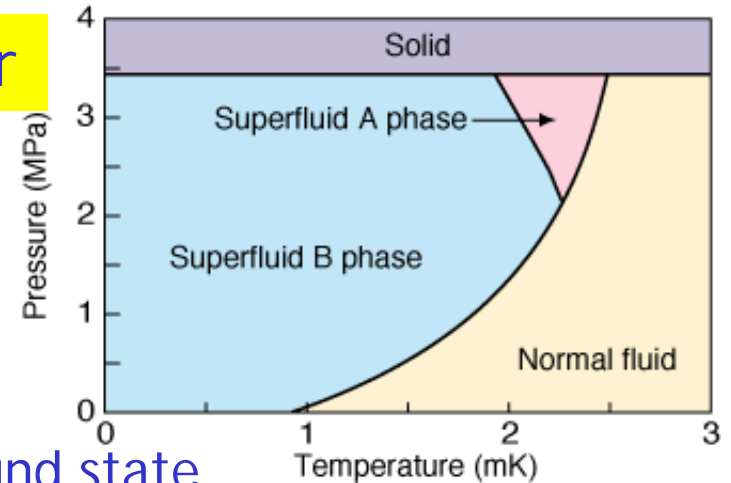
Symmetry group:

$SO(3)$



$T < T_c$

$U(1) \subset SO(3)$



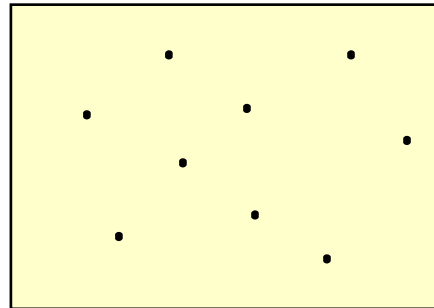
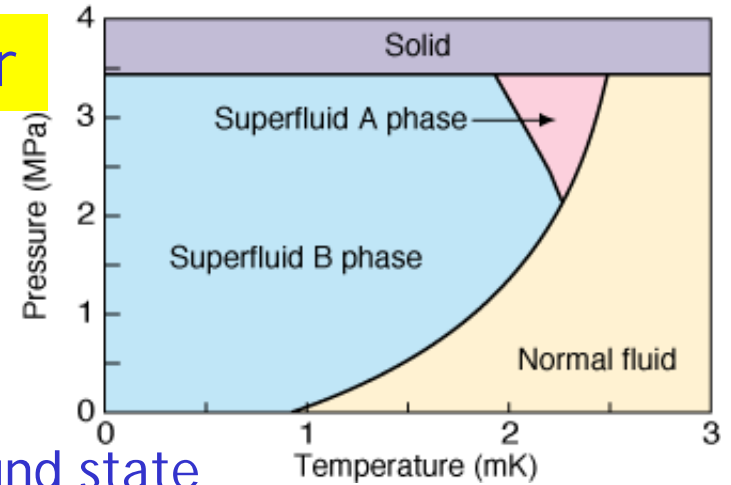
$T < T_c$ :  $SO(3)$  rotation symmetry in real space spontaneously broken

# Broken Symmetries, Long Range Order

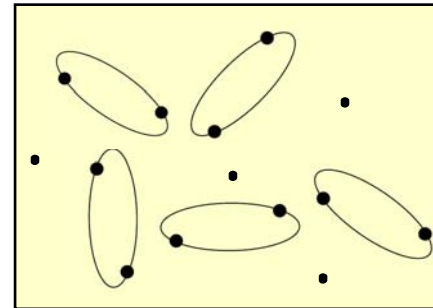
## 2. order phase transition

$T < T_c$ : higher order, lower symmetry of ground state

### III. Conventional superconductor



$T > T_c$



$T < T_c$

Pair amplitude  $\langle c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger \rangle = 0$

$\Delta e^{i\phi}$  "Order parameter"

Gauge transf.  $c_{\mathbf{k}\sigma}^\dagger \rightarrow c_{\mathbf{k}\sigma}^\dagger e^{i\varphi}$  : gauge invariant

not gauge invariant

Symmetry group U(1)

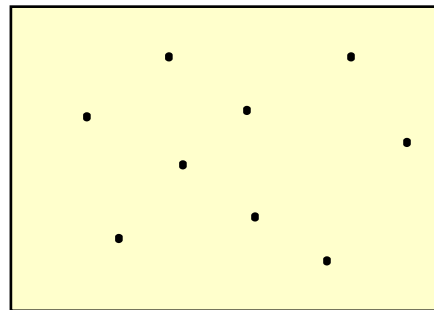
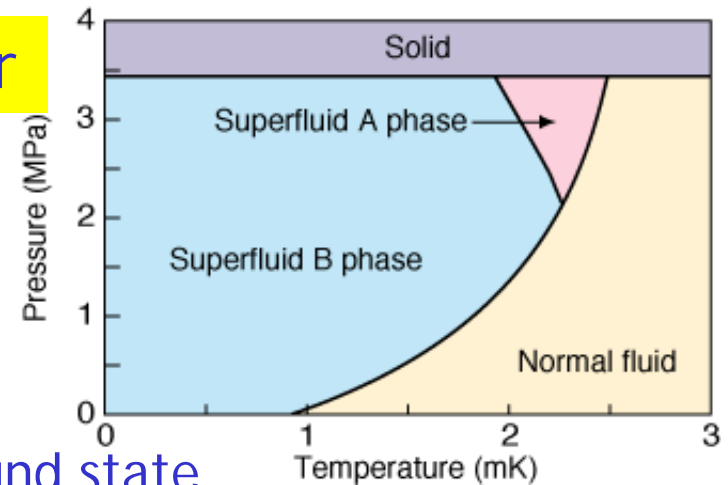
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# Broken Symmetries, Long Range Order

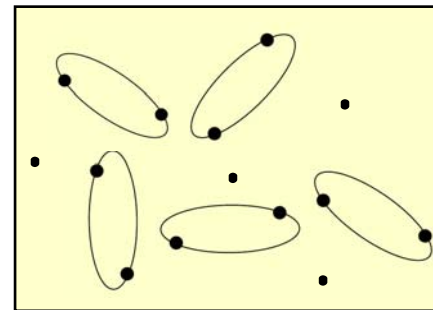
## 2. order phase transition

$T < T_c$ : higher order, lower symmetry of ground state

### III. Conventional superconductor



$T > T_c$



$T < T_c$

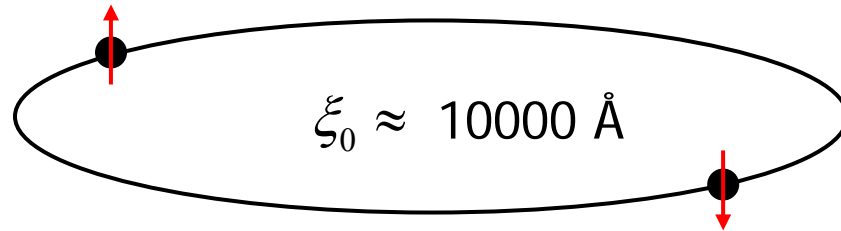
$T < T_c$ : U(1) "gauge symmetry" spontaneously broken

→ U(1) gauge symmetry also broken in BEC

# Cooper pairing of Fermions vs. Bose-Einstein condensation

Cooper pair: "Quasi-boson"

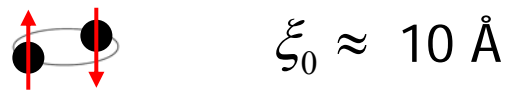
Conventional superconductors



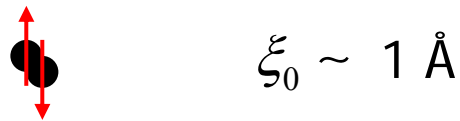
Superfluid  $^3\text{He}$



High  $T_c$  superconductors



Tightly packed bosons



BCS

Continuous crossover?

BEC

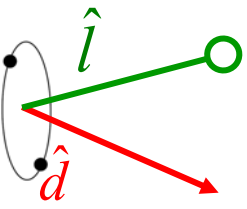
New insights from BEC of cold atoms

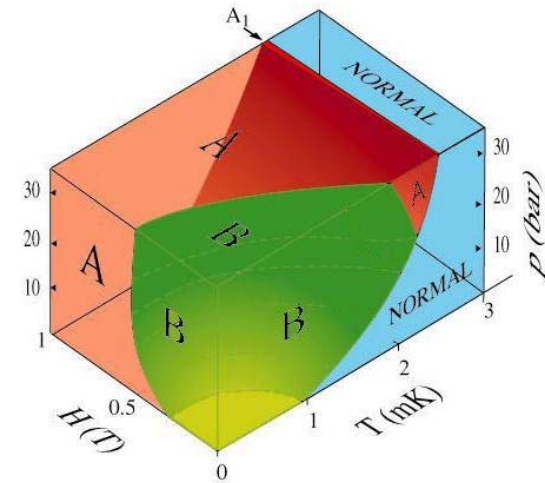
Leggett (1980)



# Broken symmetries in superfluid $^3\text{He}$

$L=1$ ,  $S=1$  in all phases

Cooper pair:  orbital part  
spin part



Quantum coherence in  $\left\{ \begin{array}{l} \text{phase} \\ \text{anisotropy direction for spin} \\ \text{anisotropy direction in real space} \end{array} \right.$  Superfluid,  
magnetic  
liquid crystal

Characterized by  $2 \times (2L + 1) \times (2S + 1) = 18$  real numbers

3x3 order parameter matrix  $A_{i\mu}$

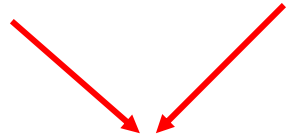
$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry spontaneously broken Leggett (1975)

# Broken symmetries in superfluid $^3\text{He}$

Mineev (1980)  
Bruder, DV (1986)

3He-B

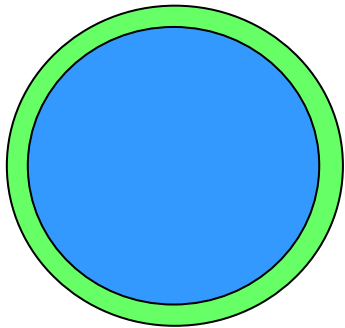
$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry broken



$\text{SO}(3)_{S+L}$

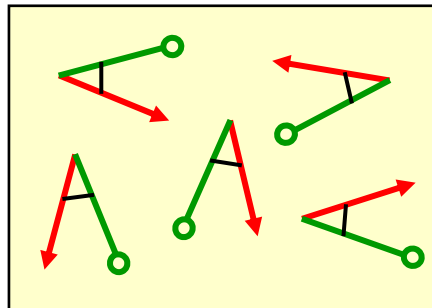


„Unconventional“ superfluidity



Spontaneously broken spin-orbit  
symmetry  
Leggett (1972)

Cooper pairs



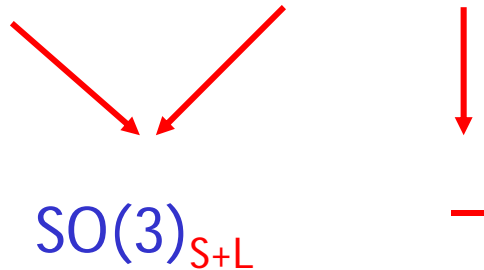
Fixed relative orientation

# Broken symmetries in superfluid $^3\text{He}$

Mineev (1980)  
Bruder, DV (1986)

3He-B

$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry broken



„Unconventional“ superfluidity

## Relation to high energy physics

Isodoublet

$\begin{pmatrix} u \\ d \end{pmatrix}_L$ ,  $\begin{pmatrix} u \\ d \end{pmatrix}_R$  chiral invariance

Global symmetry

$\text{SU}(2)_L \times \text{SU}(2)_R$

$q\bar{q}$  condensation (“Cooper pair”)

$\text{SU}(2)_{L+R}$

Goldstone excitations (bosons)

3 pions

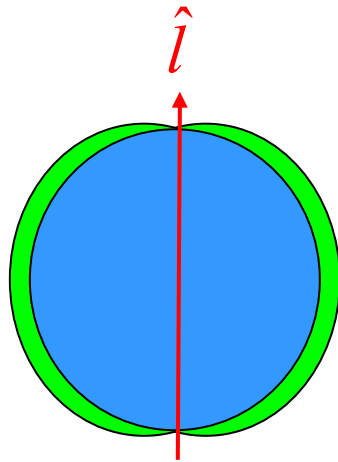
# Broken symmetries in superfluid $^3\text{He}$

Mineev (1980)  
Bruder, DV (1986)

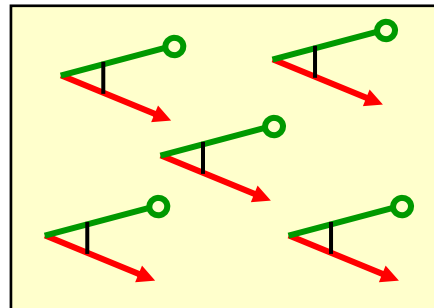
**3He-A**  $\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry broken

$\text{U}(1)_{S_z} \times \text{U}(1)_{L_z - \varphi}$

„Unconventional“ pairing

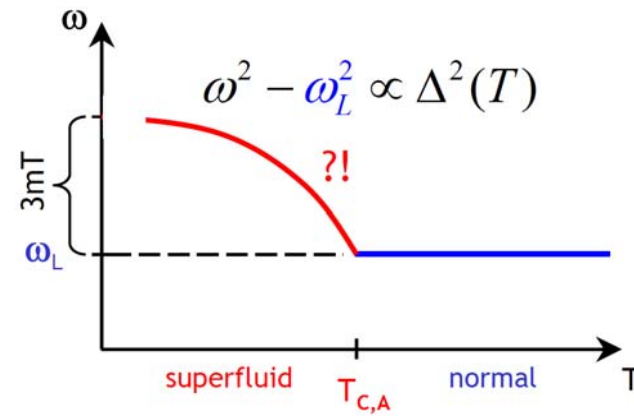


Cooper pairs



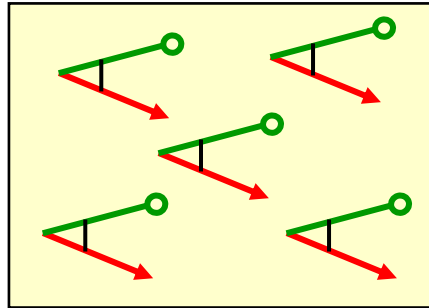
Fixed absolute orientation

# Resolution of the NMR puzzle



# Superfluid $^3\text{He}$ - a quantum amplifier

Cooper pairs in  $^3\text{He-A}$



Fixed absolute orientation

What determines the actual relative orientation of  $\hat{d}$ ,  $\hat{l}$ ?

→ Anisotropic spin-orbit interaction of nuclear dipoles:

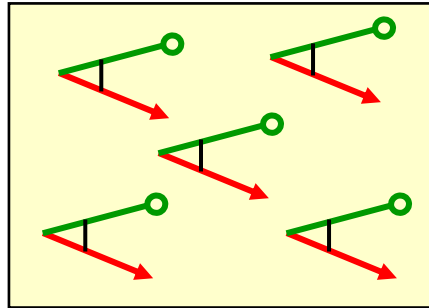


Dipole-dipole coupling of  $^3\text{He}$  nuclei:  $g_D \sim 10^{-7} K \ll T_C$

Unimportant ?!

# Superfluid $^3\text{He}$ - a quantum amplifier

Cooper pairs in  $^3\text{He-A}$



Fixed **absolute** orientation

- Long-range order in  $\hat{d}, \hat{l}$
- $g_D \sim 10^{-7} K$ : tiny, but lifts degeneracy of relative orientation

Quantum  $\Downarrow$  coherence

$\hat{d}, \hat{l}$  locked in **all** Cooper pairs

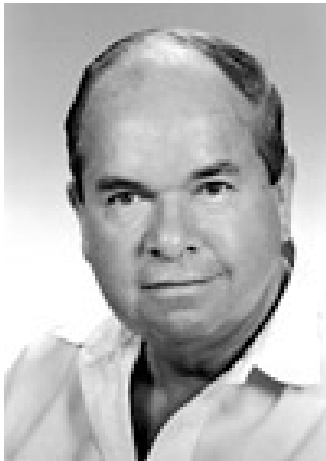


NMR frequency increases:  $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$  Leggett (1973)

→ Nuclear dipole interaction macroscopically measurable

## The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors  
and superfluids"



Alexei A. Abrikosov  
USA and Russia



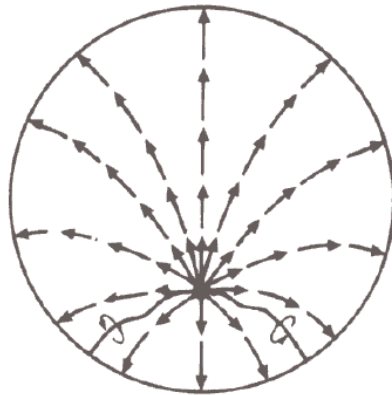
Vitaly L. Ginzburg  
Russia



Anthony J. Leggett  
UK and USA



# Order parameter textures and topological defects

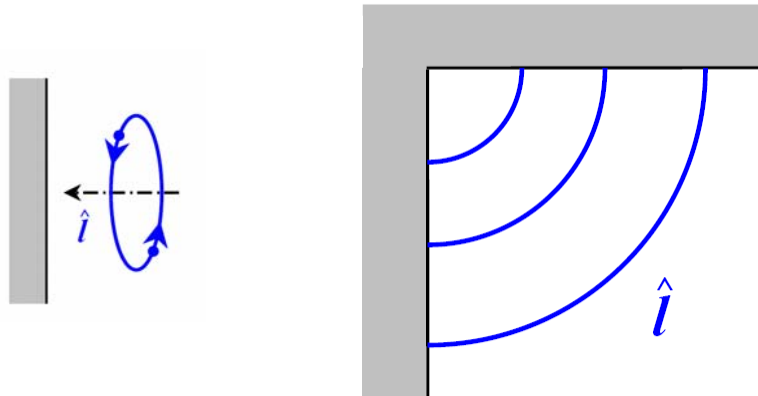


## Order parameter textures

Orientation of anisotropy directions  $\hat{d}, \hat{l}$  in  ${}^3\text{He-A}$  ?

Magnetic field  $\rightarrow \hat{d}$

Walls  $\rightarrow \hat{l}$

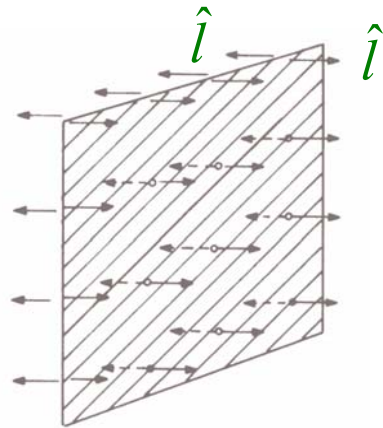


$\rightarrow$  "Textures" in  $\hat{d}, \hat{l}$   $\leftrightarrow$  liquid crystals

$\rightarrow$  Topologically stable defects: Classification by homotopy theory

# Order parameter textures and topological defects

D=2: domain walls in  $\hat{d}$  or  $\hat{l}$



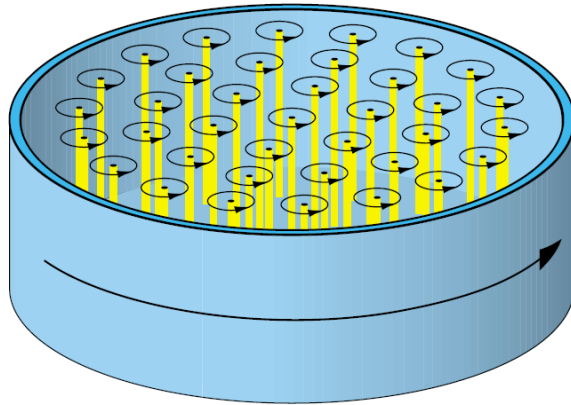
Single domain wall



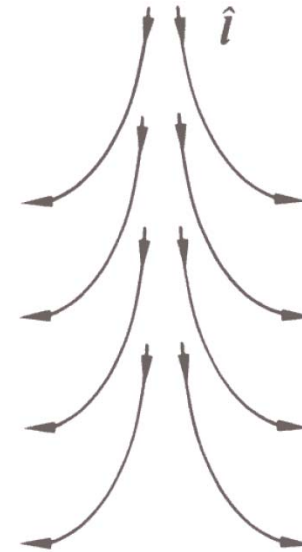
Domain wall lattice

# Order parameter textures and topological defects

D=1: Vortices



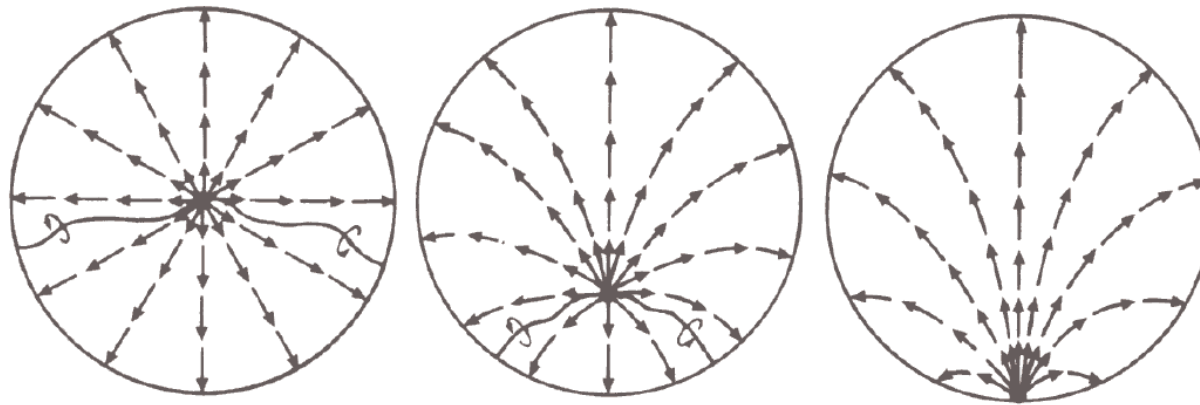
Vortex formation  
(rotation experiments)



e.g., Mermin-Ho vortex  
(non-singular)

# Order parameter textures and topological defects

D=0: Monopoles



“Boojum” in  $\hat{l}$ -texture of  $^3\text{He-A}$   
(geometric constraint)

Defect formation by, e.g.,

- rotation
- geometric constraints
- rapid crossing through phase transition

Big bang simulation  
in the low temperature lab



# Universality in continuous phase transitions



High symmetry,  
short-range order

$T > T_c$

Spins:  
para-  
magnetic

Helium:  
normal  
liquid

Universe:  
Unified forces  
and fields

$T = T_c$

Phase transition

Broken symmetry,  
long-range order

ferromagnetic

superfluid

elementary  
particles,  
fundamental  
interactions

**Defects**: domain  
walls

vortices,  
etc.

cosmic strings,  
etc. Kibble (1976)

$T < T_c$

nucleation of galaxies?

# Rapid thermal quench through 2. order phase transition

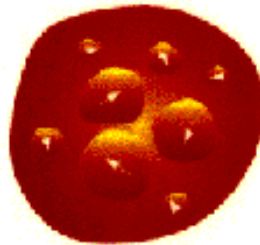
Kibble (1976)

1. Local temperature  $T \gg T_C$



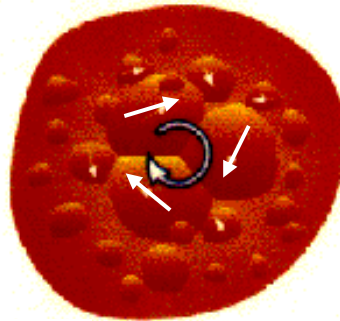
Expansion + rapid cooling

2. Nucleation of independently ordered regions

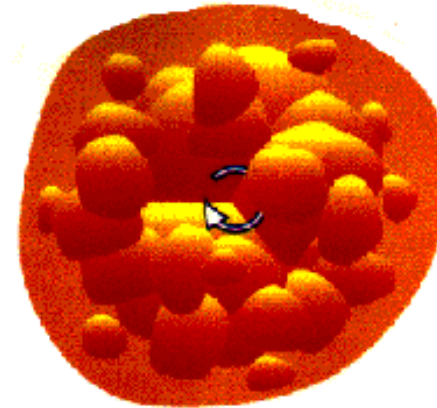


Clustering of ordered regions

→ Defects

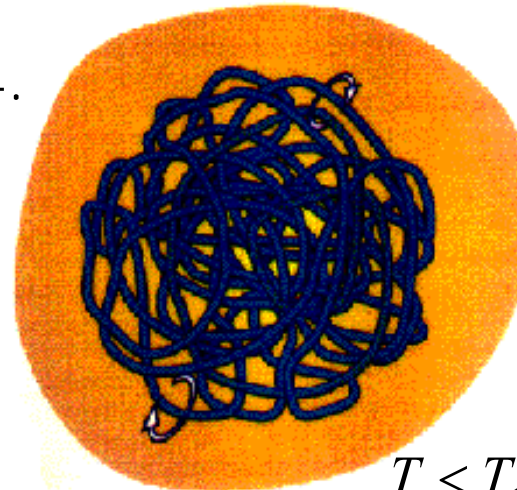


3.



Defects overlap

4.



$T < T_C$  : Vortex tangle

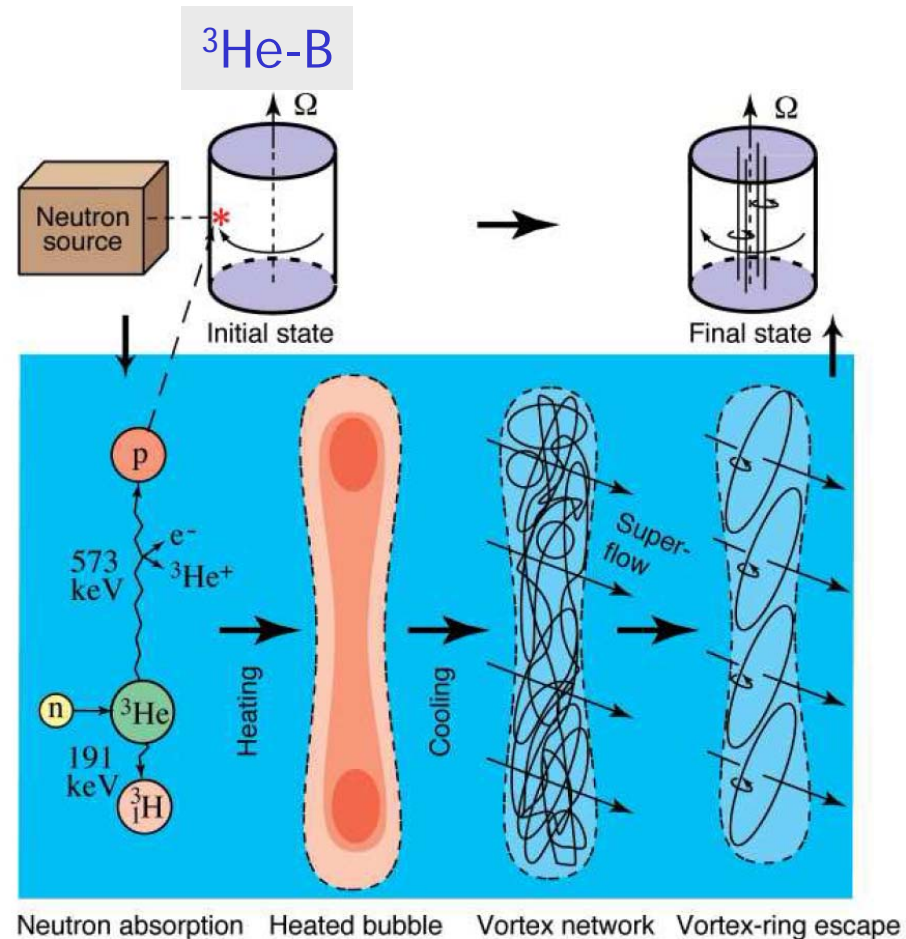
Estimate of density of defects Zurek (1985)

"Kibble-Zurek mechanism": How to test?



# Big bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)



Measured vortex tangle density:

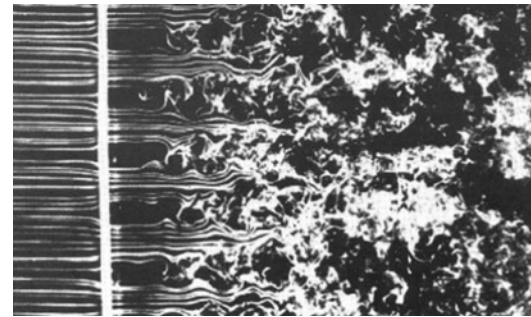
Quantitative support for Kibble-Zurek mechanism

# Present research on superfluid $^3\text{He}$ : Quantum Turbulence

## Classical Turbulence



Leonardo da Vinci (1452-1519)



Flow through grid

Quantum Turbulence = Turbulence in the absence of viscous dissipation (superfluid at  $T \rightarrow 0$ )

- Why are quantum and classical turbulence so similar?
- What provides dissipation in the absence of friction?

Test system:  $^3\text{He-B}$

Vinen, Donnelly: *Physics Today* (April, 2007)

# Conclusion

## Superfluid Helium-3:

- Anisotropic superfluid
  - 3 different bulk phases
  - Cooper pairs with internal structure
- Large symmetry group broken
  - Close connections to particle theory
  - Zoo of topological defects
  - Kibble-Zurek mechanism quantitatively verified

