

Center for  
Electronic Correlations and Magnetism  
University of Augsburg

# Superfluid Helium-3: From very low Temperatures to the Big Bang

Universität Frankfurt; May 30, 2007

Dieter Vollhardt

## Contents:

- The quantum liquids  $^3\text{He}$  and  $^4\text{He}$
- Superfluid phases of  $^3\text{He}$
- Broken symmetries and long-range order
- Topological defects
- Big Bang simulation in the low temperature lab

# Helium

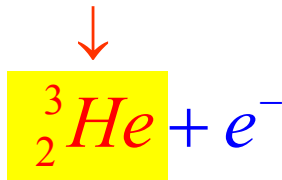
Two stable Helium isotopes:

$^4\text{He}$ : air, oil wells, ...

Janssen/Lockyer (1868)

Ramsay (1895)

$^3\text{He}$ :  $^6_3\text{Li} + ^1_0n \rightarrow ^3_1\text{H} + \alpha$  (1939)



$\frac{\text{He}}{\text{Luft}} \approx 5 \times 10^{-6}, \quad \frac{^3\text{He}}{^4\text{He}} \Big _{\text{Luft}} \approx 1 \times 10^{-6}$
--

Research on macroscopic samples of  $^3\text{He}$  since 1947

# Helium

Atoms: spherical, hard core diameter  $\sim 2.5 \text{ \AA}$

Interaction: 

- hard sphere **repulsion**
- van der Waals dipole (+ ...) **attraction**

Boiling point: 4.2 K,  $^4\text{He}$  Kamerlingh Onnes (1908)



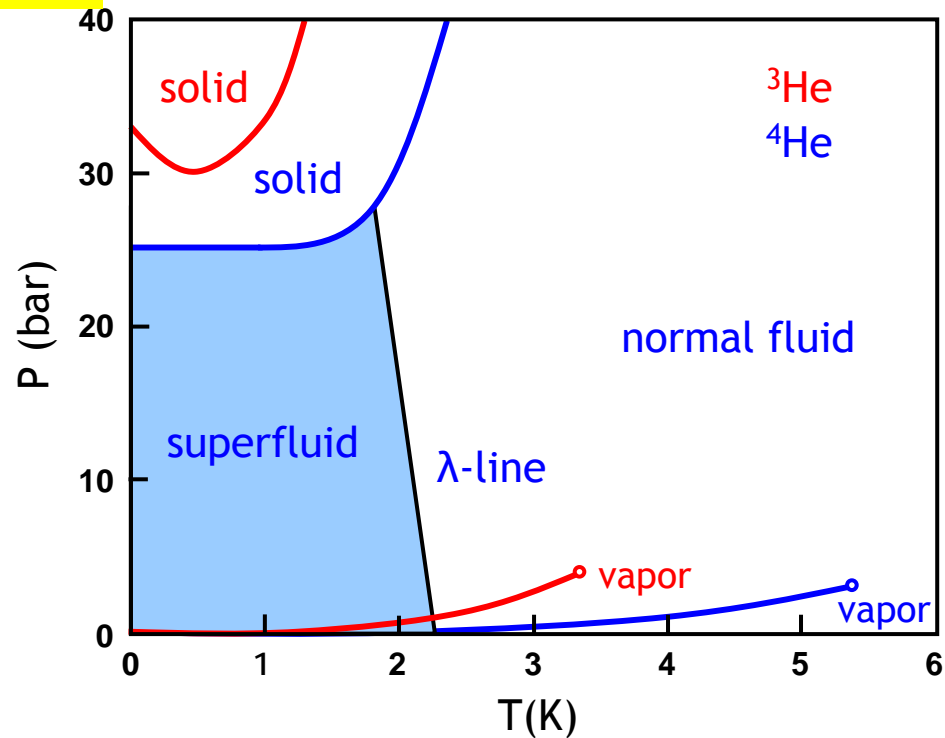
Nobel Prize 1913

3.2 K,  $^3\text{He}$  Sydoriak, *et al.* (1949)

Dense, simple liquid { 

- isotropic
- short-range interactions
- extremely pure
- nuclear spin  $S=1/2$

# Helium

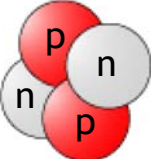
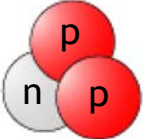


- Atoms:
- spherical shape  $\rightarrow$  weak attraction
  - light mass  $\rightarrow$  strong zero-point motion

$T \rightarrow 0$ ,  $P \lesssim 30$  bar: Helium remains liquid

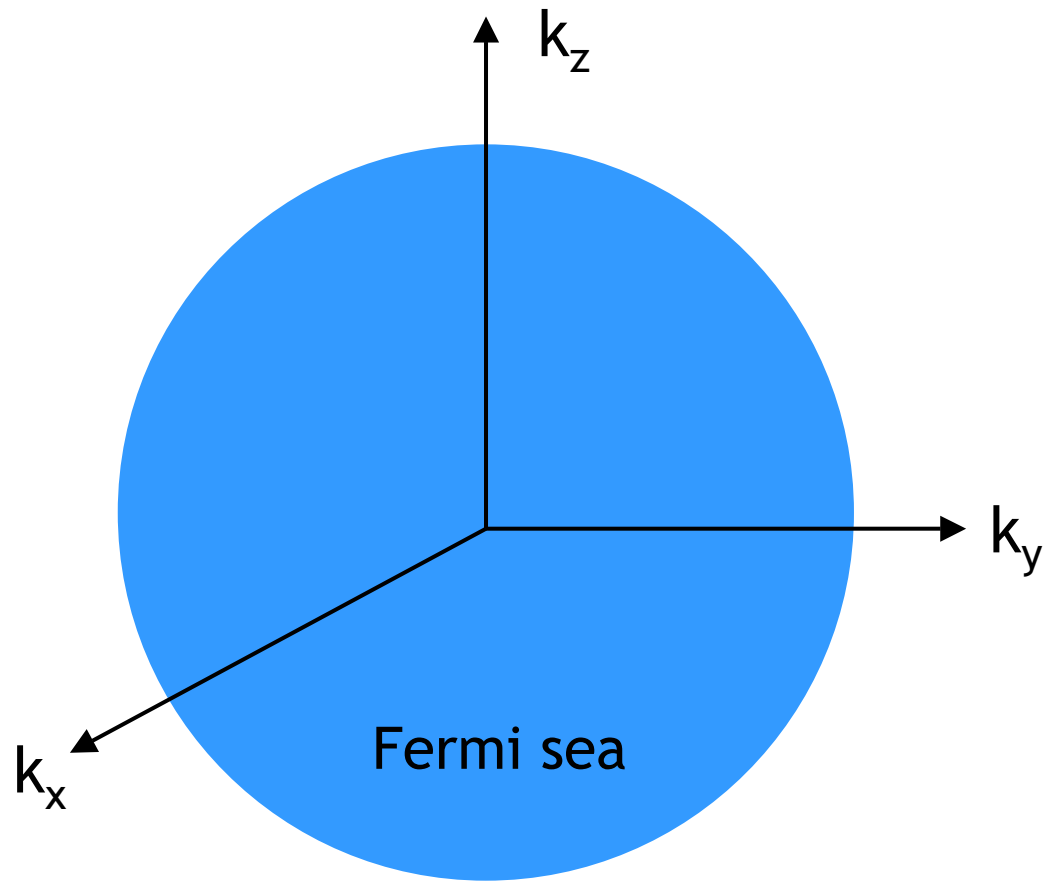
$$\lambda \propto \frac{\hbar}{\sqrt{k_B T}} \xrightarrow{T \rightarrow 0} \text{Macroscopic quantum phenomena}$$

# Helium

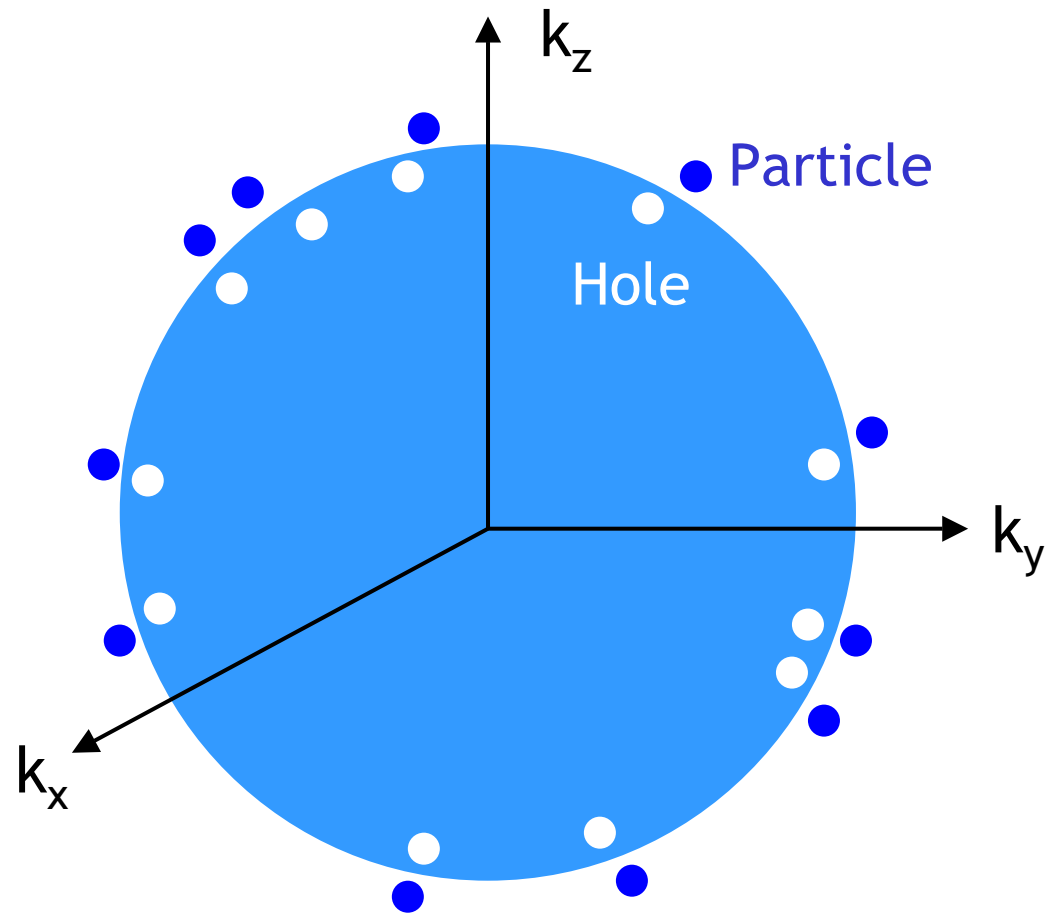
	$^4\text{He}$	$^3\text{He}$
Electron shell:	$2 e^-, S = 0$	
Nucleus:	 $S = 0$	 $S = \frac{1}{2}\hbar$
Atom(!) is a	Boson	Fermion
	$T_\lambda = 2.2 \text{ K}$ ("BEC")	$T_c = ???$
		Fermi liquid theory

Quantum liquids

Fermi gas: Ground state



# Fermi gas: Excited states ( $T > 0$ )

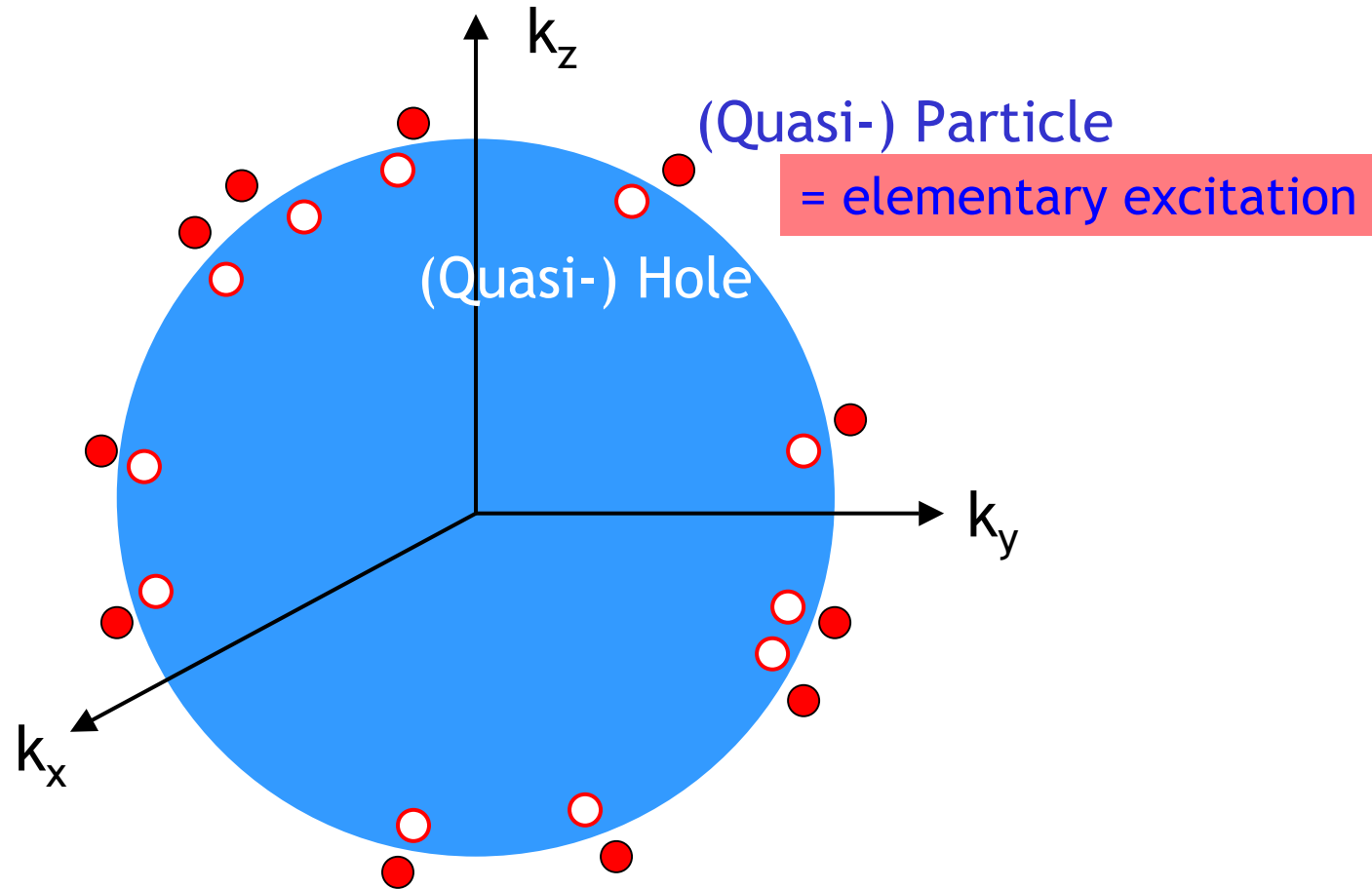


Switch on interaction adiabatically



# Landau Fermi liquid

1-1 correspondence  
between states

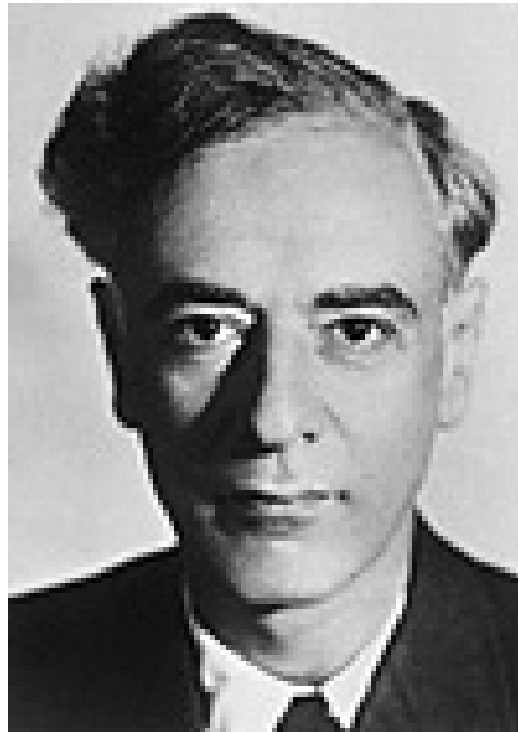


Prototype: Helium-3

- Large effective mass
- Strongly enhanced spin susceptibility
- Strongly reduced compressibility

## The Nobel Prize in Physics 1962

"for his pioneering theories for condensed matter,  
especially liquid helium"

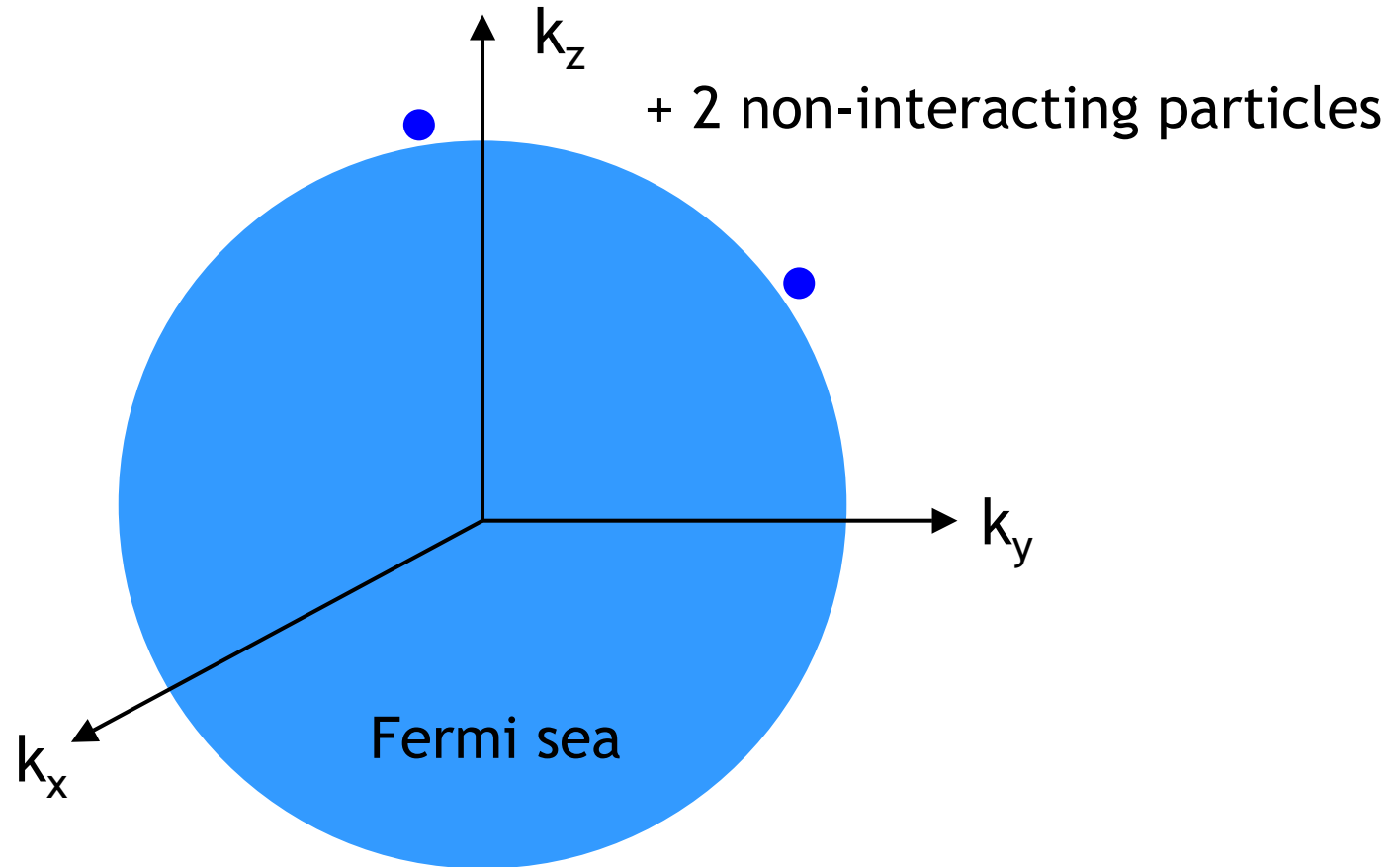


Lev Davidovich Landau

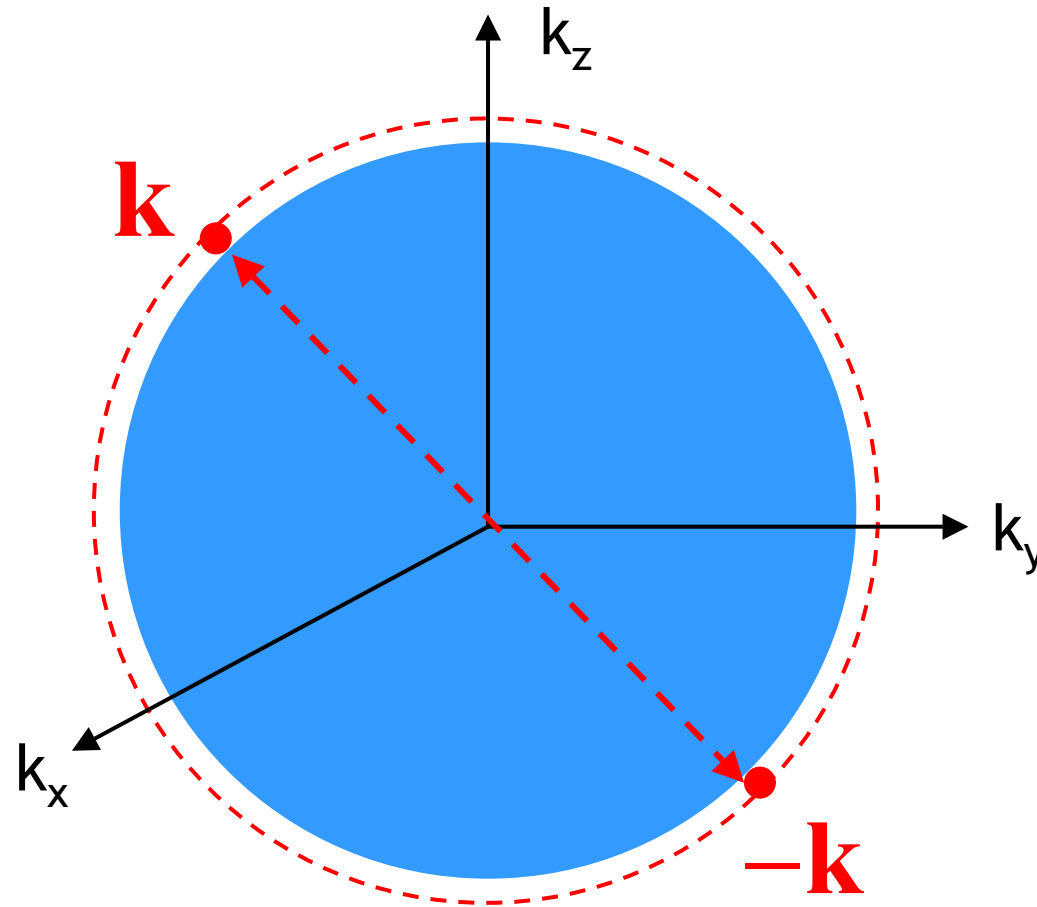
USSR

b. 1908, d. 1968

# Instability of Landau Fermi liquid

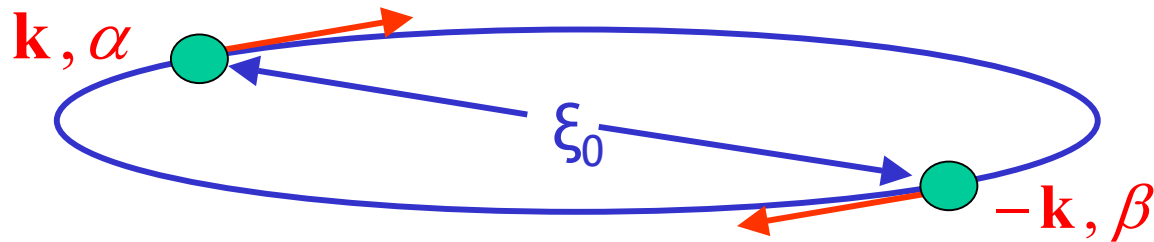


Arbitrarily weak attraction  $\Rightarrow$  Cooper instability



Universal fermionic property

Arbitrarily weak attraction  $\Rightarrow$  Cooper pair  $(\mathbf{k}, \alpha; -\mathbf{k}, \beta)$



$$\Psi_{L=0,2,4,\dots} = \psi(\mathbf{r}) |\uparrow\downarrow - \downarrow\uparrow\rangle$$

**S=0 (singlet)**

$$\begin{aligned} \Psi_{L=1,3,5,\dots} = & \psi_+(\mathbf{r}) |\uparrow\uparrow\rangle \\ & + \psi_0(\mathbf{r}) |\uparrow\downarrow + \downarrow\uparrow\rangle \\ & + \psi_-(\mathbf{r}) |\downarrow\downarrow\rangle \end{aligned}$$

**S=1 (triplet)**

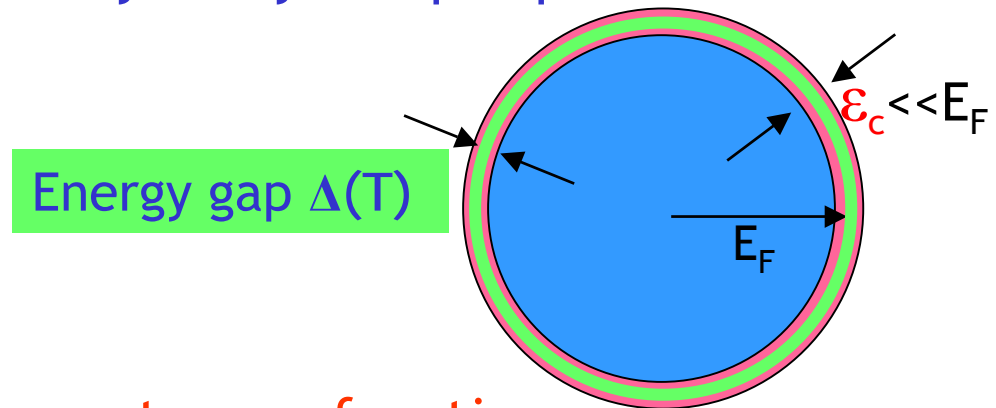
**L = 0: isotropic wave function**  
**L > 0: anisotropic wave function**

Helium-3: Strongly repulsive interaction  $\rightarrow$  **L>0** expected

# BCS theory

Bardeen, Cooper, Schrieffer (1957)

Generalization to macroscopically many Cooper pairs



→ "Pair condensate"  
with macroscopically coherent wave function

Transition temperature

$$T_c = 1.13 \epsilon_c \exp(-1 / N(0) |V_L|)$$

"weak coupling theory"

$\epsilon_c, V_L$ : Magnitude ? Origin ( $e^- + ?$ ) ? →  $T_c$  ?

Thanksgiving 1971: Transition in  $^3\text{He}$  at  $T_c = 0.0026 \text{ K}$

Osheroff, Richardson, Lee (1972)

**The Nobel Prize in Physics 1996**  
"for their discovery of superfluidity in helium-3"



David M. Lee  
USA, Cornell  
b. 1931



Douglas D.  
Osheroff USA,  
Stanford  
b. 1945



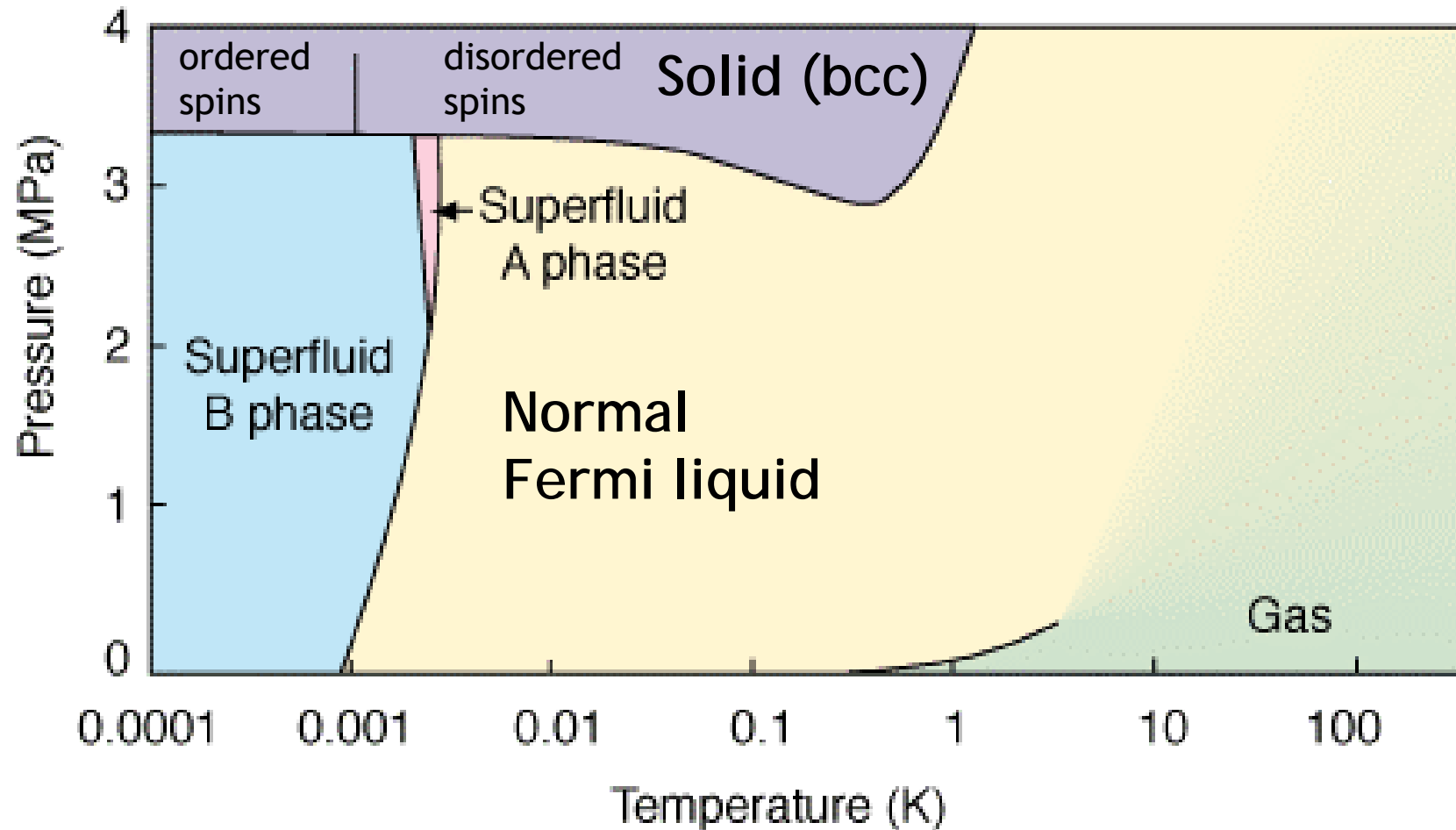
Robert C.  
Richardson  
USA, Cornell  
b. 1937

# Phase diagram of Helium-3

P-T phase diagram

Dense, simple liquid

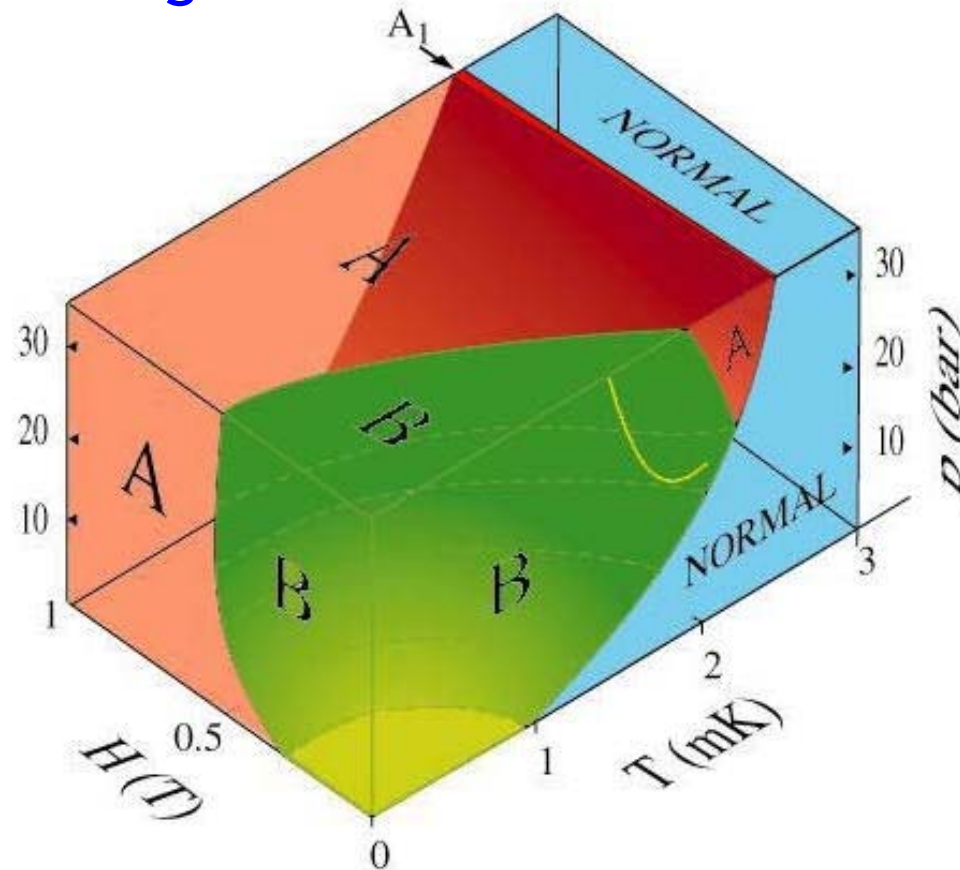
- isotropic
- short-range interactions
- extremely pure
- nuclear spin  $S=1/2$





# Phase diagram of Helium-3

## P-T-H phase diagram



“Very low temperatures”:  $T \ll T_{\text{boiling}} \sim 3\text{-}4\text{ K}$   
 $\ll T_{\text{backgr. rad.}} \sim 3\text{ K}$

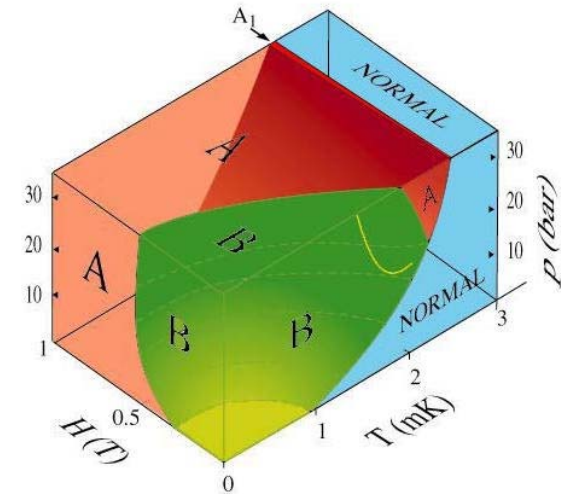
# Superfluid phases of $^3\text{He}$

Theory + experiment:  $L=1$ ,  $S=1$  in all phases

Leggett

Wölfle

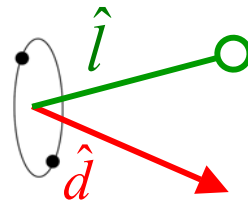
Mermin, ...



Pairing mechanism: Spin fluctuations

Anderson, Brinkman (1973)

→ anisotropy directions  
in a  $^3\text{He}$  Cooper pair



orbital part

spin part

Anisotropy + quantum coherence

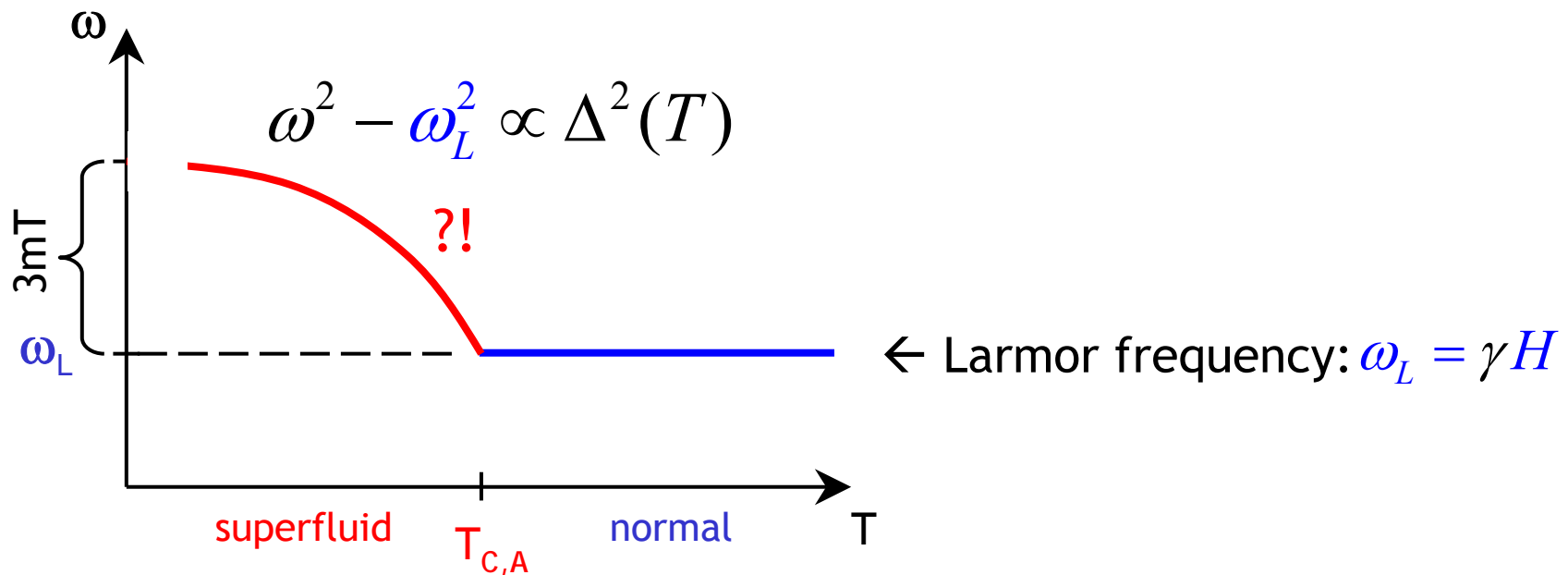


Long-range order in real and spin space

... and a mystery!

NMR experiment on nuclear spins  $I = \frac{1}{2}\hbar$

Osheroff *et al.* (1972)



Shift of  $\omega_L$   $\Leftrightarrow$  spin-nonconserving interactions  
 $\rightarrow$  nuclear dipole interaction  $g_D \sim 10^{-7} K \ll T_C$

Origin of frequency shift ?!

Leggett (1973)

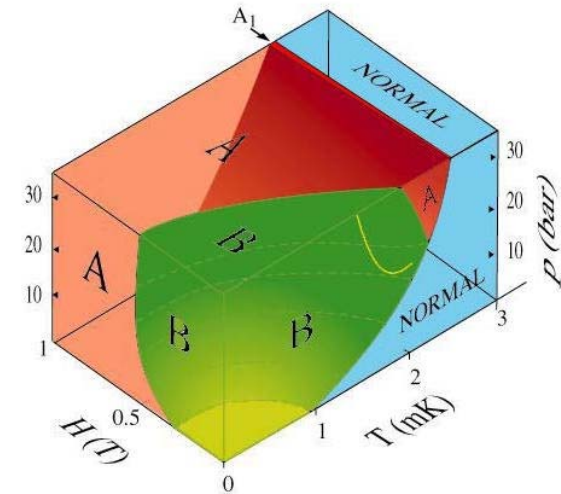
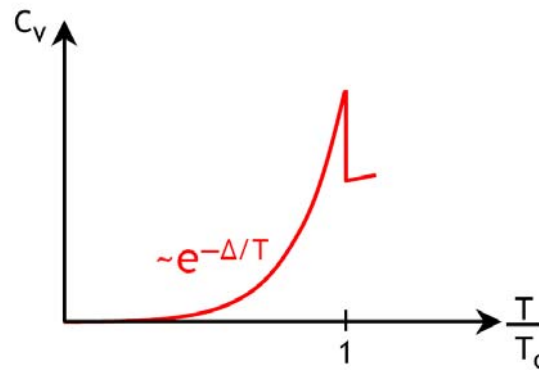
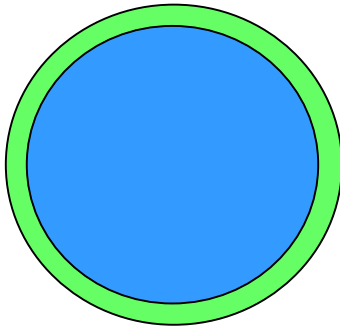
# The superfluid phases of $^3\text{He}$

# B-phase

$$\Psi = |\uparrow\uparrow\rangle + |\uparrow\downarrow + \downarrow\uparrow\rangle + |\downarrow\downarrow\rangle$$

$$\Delta(\mathbf{k}) = \Delta_0$$

Balian, Werthamer (1963)  
Vdovin (1963)



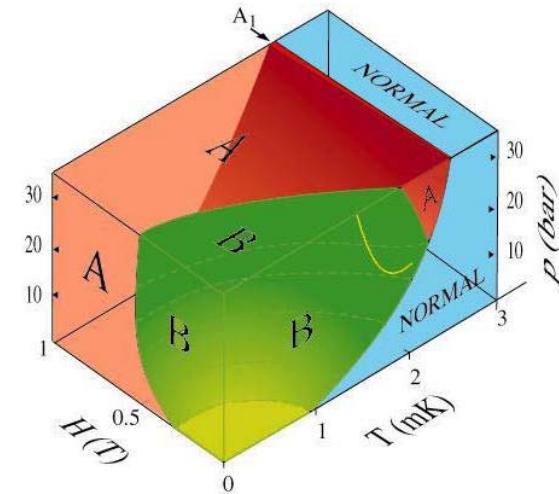
(pseudo-) isotropic state  $\leftrightarrow$  s-wave superconductor

Weak-coupling theory: stable for all  $T < T_c$

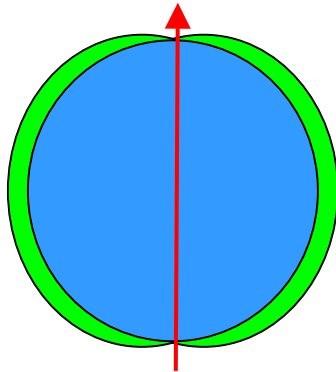
## A-phase

$$\Psi = |\uparrow\uparrow\rangle + |\downarrow\downarrow\rangle \rightarrow \text{strong anisotropy}$$

$$\Delta(\hat{k}) = \Delta_0 \sin(\hat{k}, \hat{l}) \quad \text{Anderson, Morel (1961)}$$



$\hat{l}$  Cooper pair  
orbital angular momentum



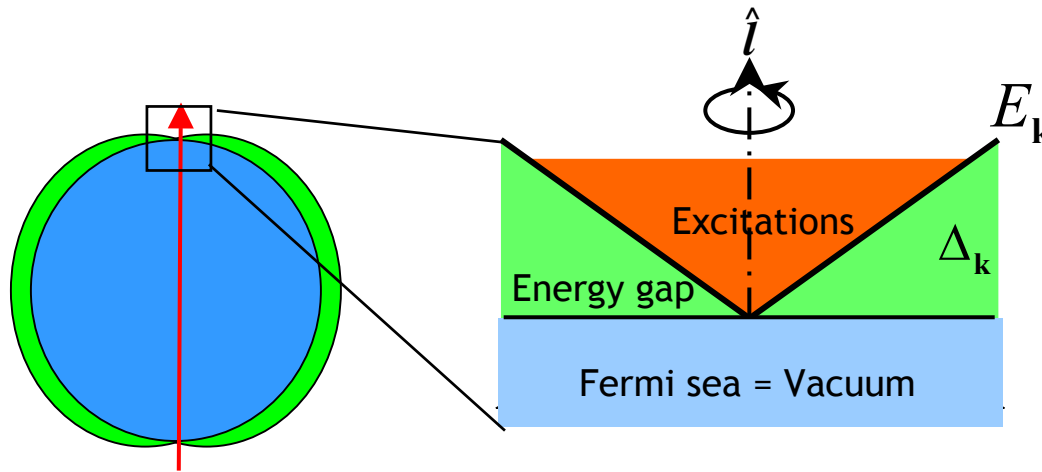
*Axial state* has point nodes

→ “unconventional” pairing in  
heavy fermion/high- $T_c$  superconductors

Strong-coupling effect

# <sup>3</sup>He-A: Spectrum near poles

Volovik (1987)



$$E_{\mathbf{k}}^2 = v_F^2 (k - k_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \end{cases} \quad \text{2 chiralities}$$

$$g^{ij} = v_F^2 l_i l_j + \left( \frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

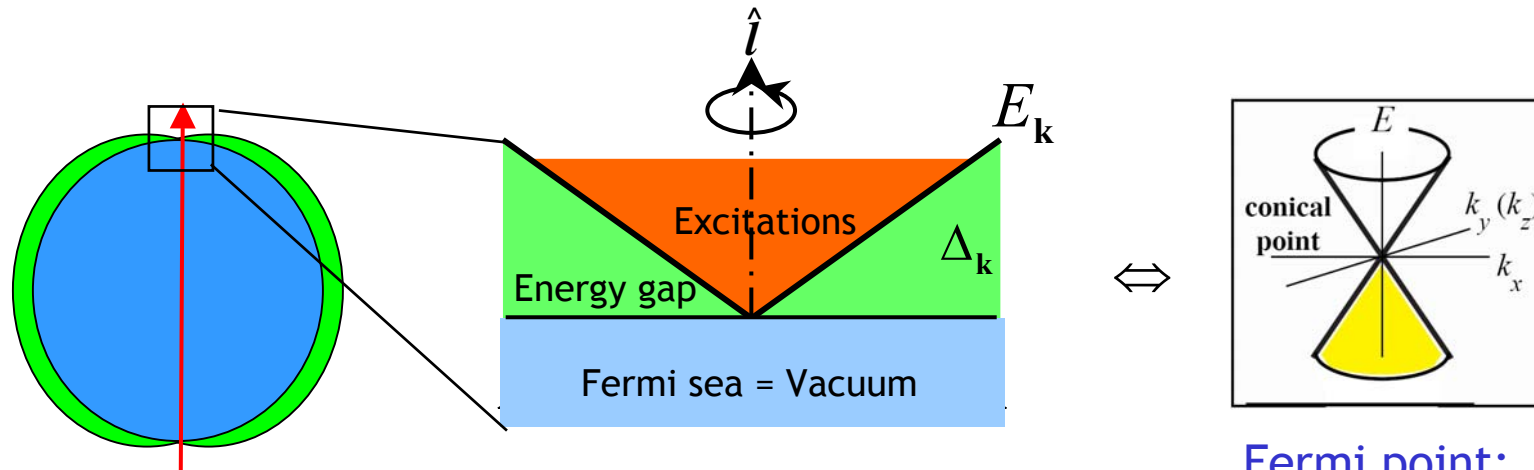
$$\mathbf{A} = k_F \hat{\mathbf{l}}$$

$$\mathbf{p} = \mathbf{k} - e\mathbf{A}$$

Lorentz invariance:  
Symmetry enhancement  
at low energies

# <sup>3</sup>He-A: Spectrum near poles

Volovik (1987)



Fermi point:  
spectral flow

$$E_{\mathbf{k}}^2 = v_F^2 (\mathbf{k} - \mathbf{k}_F)^2 + \Delta_0^2 \sin^2(\hat{\mathbf{k}}, \hat{\mathbf{l}}) = g^{ij} p_i p_j$$

$$e = \begin{cases} +1 & \hat{\mathbf{k}} \parallel +\hat{\mathbf{l}} \\ -1 & \hat{\mathbf{k}} \parallel -\hat{\mathbf{l}} \end{cases} \quad \text{2 chiralities}$$

$$g^{ij} = v_F^2 l_i l_j + \left( \frac{\Delta}{k_F} \right)^2 (\delta_{ij} - l_i l_j)$$

⇔ Massless, chiral leptons, e.g., neutrino  $E(\mathbf{p}) = cp$

→ Chiral anomaly of standard model

*The Universe in a Helium Droplet,*  
Volovik (2003)

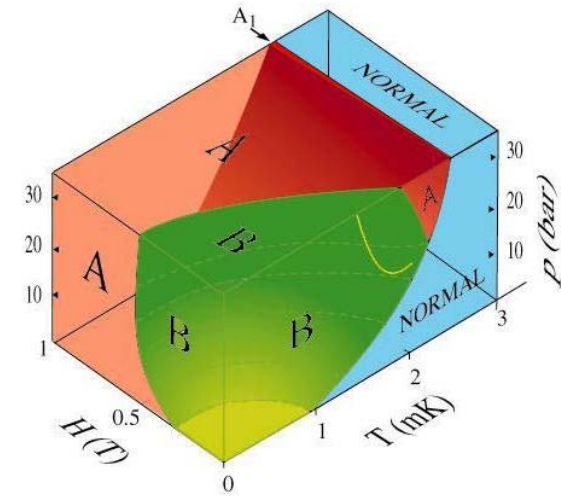


**A<sub>1</sub>-phase**

finite magnetic field

$$\Psi = |\uparrow\uparrow\rangle$$

Long-range ordered magnetic liquid

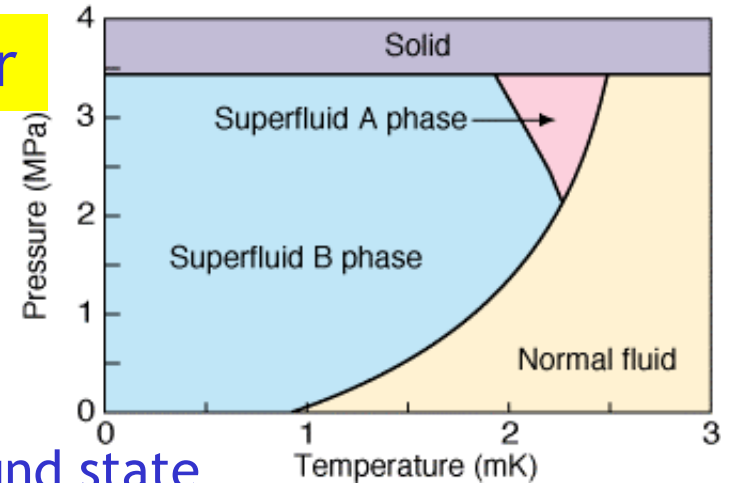


# Broken Symmetries, Long Range Order

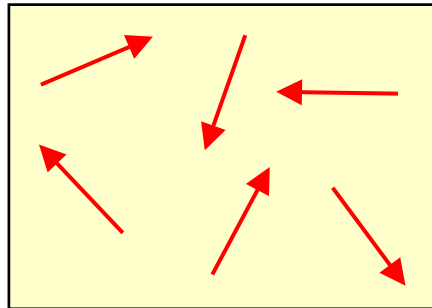
# Broken Symmetries, Long Range Order

Normal  $^3\text{He} \leftrightarrow ^3\text{He-A}, ^3\text{He-B}$ :  
2. order phase transition

$T < T_c$ : higher order, lower symmetry of ground state



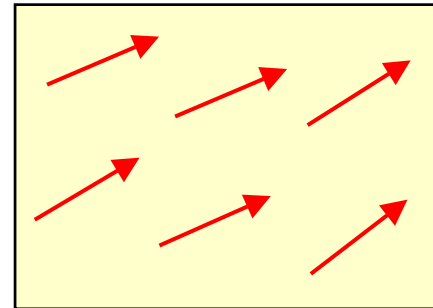
## I. Ferromagnet



$T > T_c$

Average magnetization:  
Symmetry group:

$\langle \mathbf{M} \rangle = 0$   
 $SO(3)$



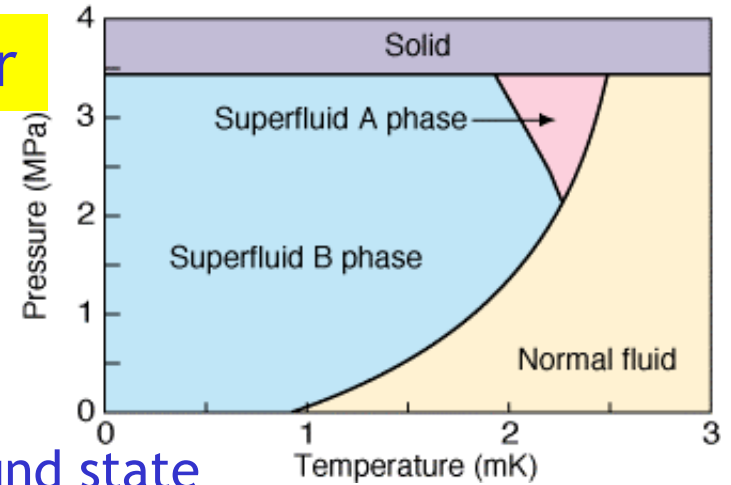
$T < T_c$

$\langle \mathbf{M} \rangle \neq 0$  Order parameter  
 $U(1) \subset SO(3)$

$T < T_c$ :  $SO(3)$  rotation symmetry in spin space spontaneously broken

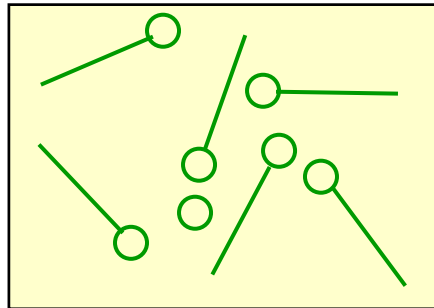
# Broken Symmetries, Long Range Order

## 2. order phase transition

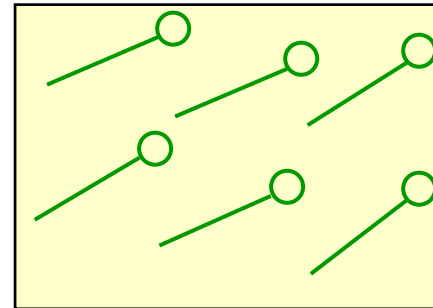


$T < T_c$ : higher order, lower symmetry of ground state

## II. Liquid crystal



$T > T_c$   
 $SO(3)$



$T < T_c$   
 $U(1) \subset SO(3)$

Symmetry group:

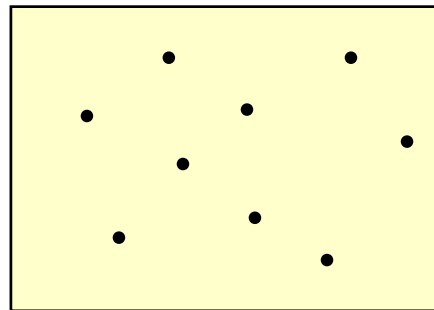
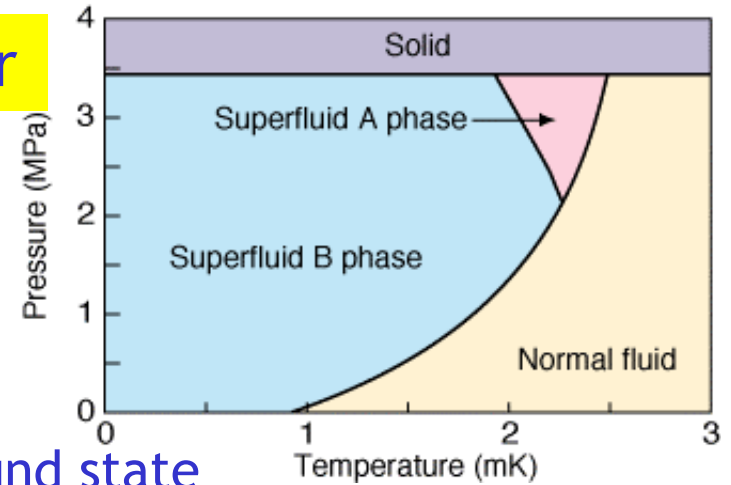
$T < T_c$ :  $SO(3)$  rotation symmetry in real space spontaneously broken

# Broken Symmetries, Long Range Order

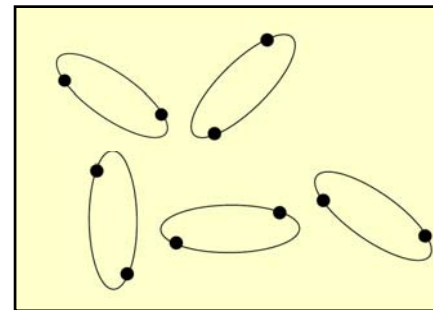
## 2. order phase transition

$T < T_c$ : higher order, lower symmetry of ground state

### III. Conventional superconductor



$T > T_c$



$T < T_c$

Pair amplitude  $\langle c_{\mathbf{k}\uparrow}^\dagger c_{-\mathbf{k}\downarrow}^\dagger \rangle = 0$

$\Delta e^{i\phi}$  Order parameter

Gauge transf.

$$c_{\mathbf{k}\sigma}^\dagger \rightarrow c_{\mathbf{k}\sigma}^\dagger e^{i\phi}$$

gauge invariant

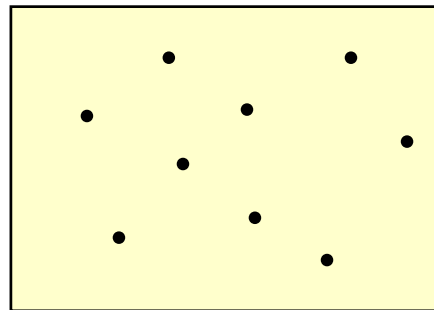
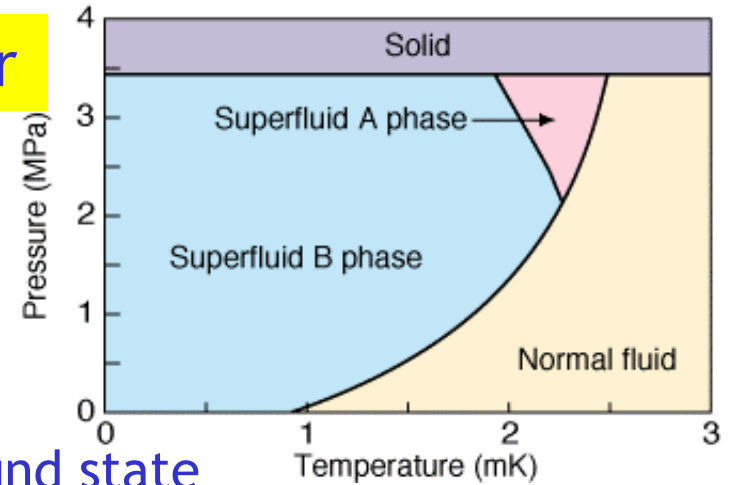
not gauge invariant

# Broken Symmetries, Long Range Order

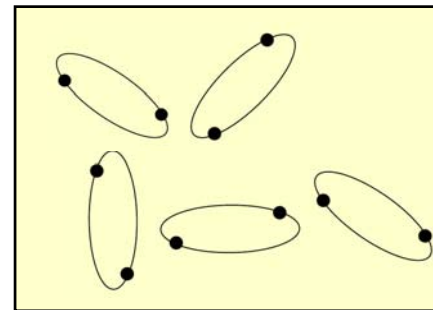
## 2. order phase transition

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$\Delta e^{i\phi}$  Order parameter

Gauge transf.

$$c_{\mathbf{k}\sigma}^\dagger \rightarrow c_{\mathbf{k}\sigma}^\dagger e^{i\phi}$$

Symmetry group:

U(1)

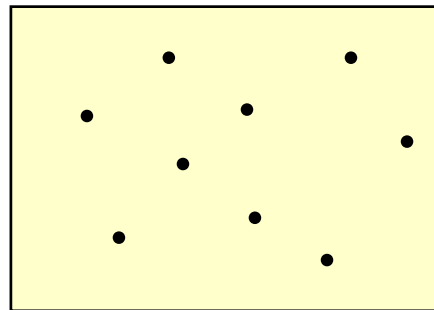
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# Broken Symmetries, Long Range Order

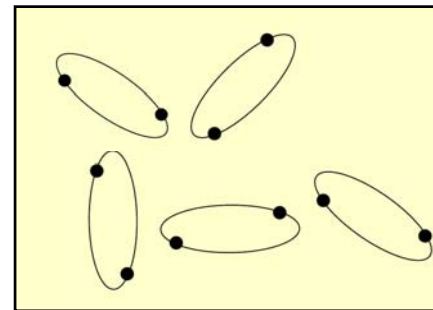
## 2. order phase transition

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### III. Conventional superconductor



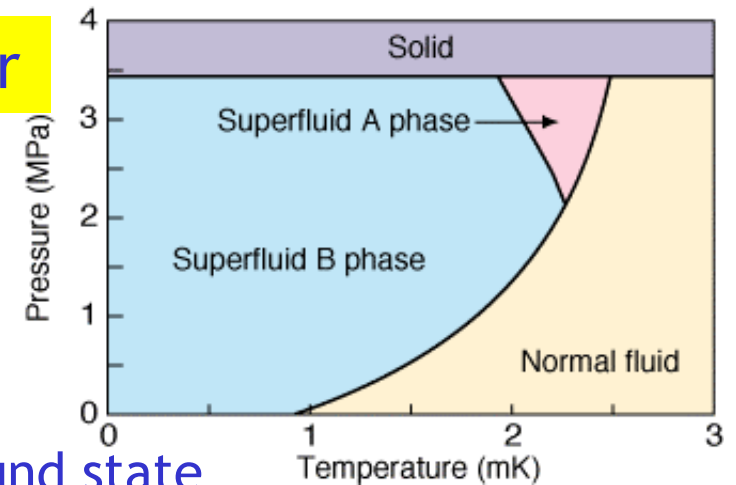
$T > T_c$



$T < T_c$

$T < T_c$ : U(1) “gauge symmetry” spontaneously broken

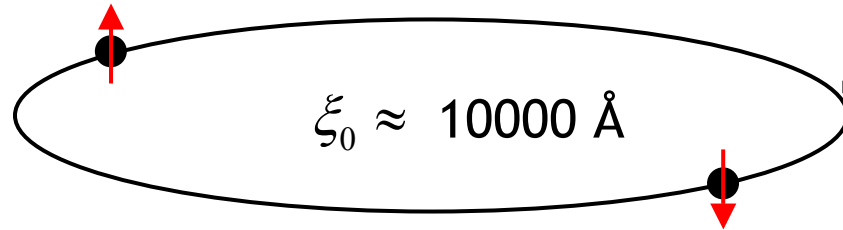
Conventional superconductivity (BCS)  $\leftrightarrow$  Superfluid  $^4\text{He}$  (BEC)



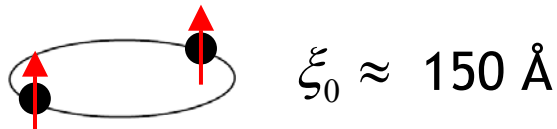
# Cooper pairing of Fermions vs. Bose-Einstein condensation

Cooper pair: "Quasi-boson"

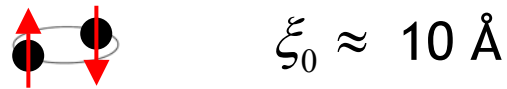
Conventional superconductors



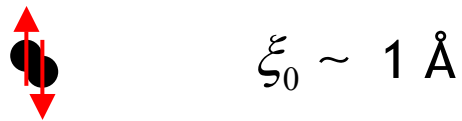
Superfluid  $^3\text{He}$



High  $T_c$  superconductors



Tightly packed bosons



BCS

Continuous crossover?

BEC

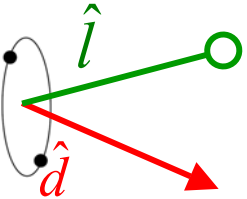
New insights from BEC of cold atoms

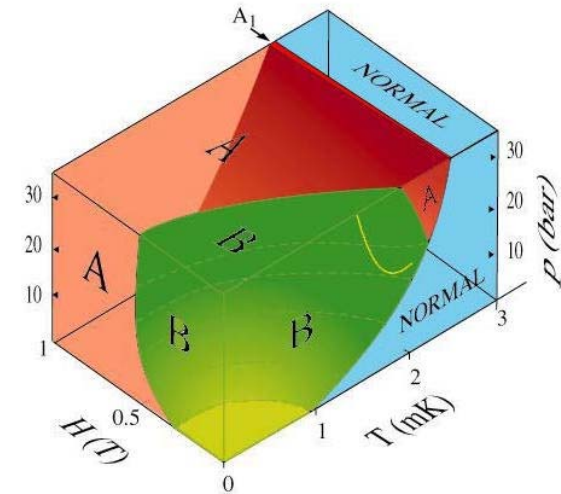
Leggett (1980)



# Broken symmetries in superfluid $^3\text{He}$

$L=1$ ,  $S=1$  in all phases

Cooper pair:  orbital part  
spin part



Quantum coherence in  $\left\{ \begin{array}{l} \text{phase} \\ \text{anisotropy direction for spin} \\ \text{anisotropy direction in real space} \end{array} \right.$  Superfluid,  
magnetic  
liquid crystal

Characterized by  $2 \times (2L + 1) \times (2S + 1) = 18$  real numbers

3x3 order parameter matrix  $A_{j\mu}$

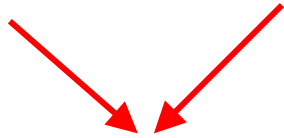
$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry spontaneously broken Leggett (1975)

# Broken symmetries in superfluid $^3\text{He}$

Mineev (1980)  
Bruder, DV (1986)

3He-B

$\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry broken

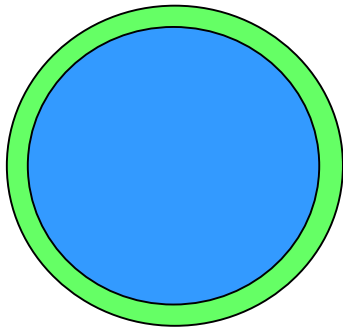


$\text{SO}(3)_{S+L}$



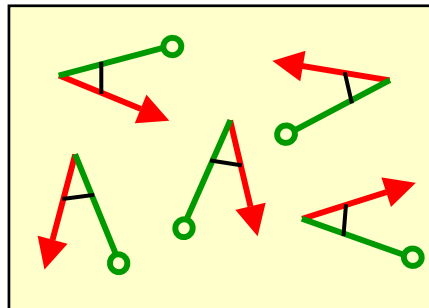
—

„Unconventional“ pairing



“Spontaneously broken spin-orbit  
Symmetry“  
Leggett (1972)

Cooper pairs

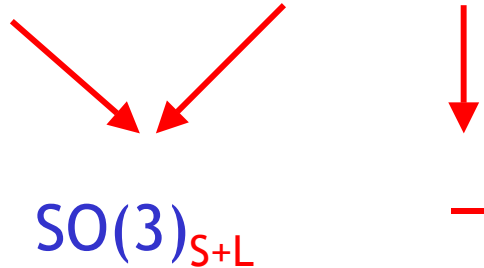


Fixed relative orientation

# Broken symmetries in superfluid $^3\text{He}$

Mineev (1980)  
Bruder, DV (1986)

**3He-B**  $\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry broken



## Relation to high energy physics

Isodoublet

$\begin{pmatrix} u \\ d \end{pmatrix}_L$ ,  $\begin{pmatrix} u \\ d \end{pmatrix}_R$  chiral invariance

Global symmetry

$\text{SU}(2)_L \times \text{SU}(2)_R$



Goldstone excitations (bosons)

3 pions

# Broken symmetries in superfluid $^3\text{He}$

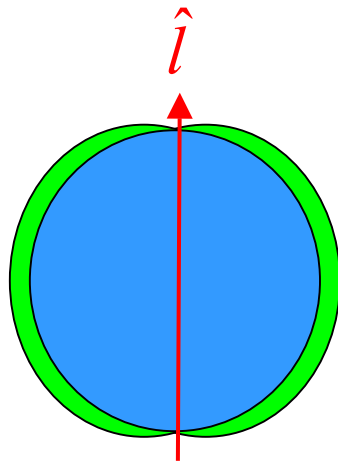
Mineev (1980)  
Bruder, DV (1986)

$^3\text{He-A}$   $\text{SO}(3)_S \times \text{SO}(3)_L \times \text{U}(1)_\varphi$  symmetry broken

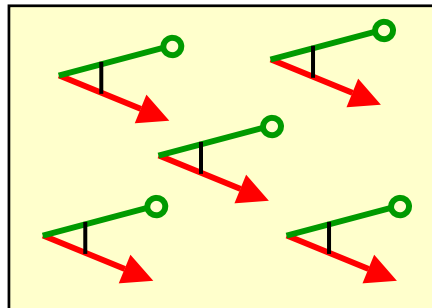
$\text{U}(1)_{S_z} \times \text{U}(1)_{L_z - \varphi}$

„Unconventional“ pairing

→ heavy fermion/high- $T_c$  superconductivity



Cooper pairs

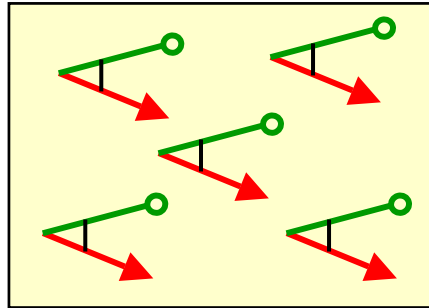


Fixed absolute orientation

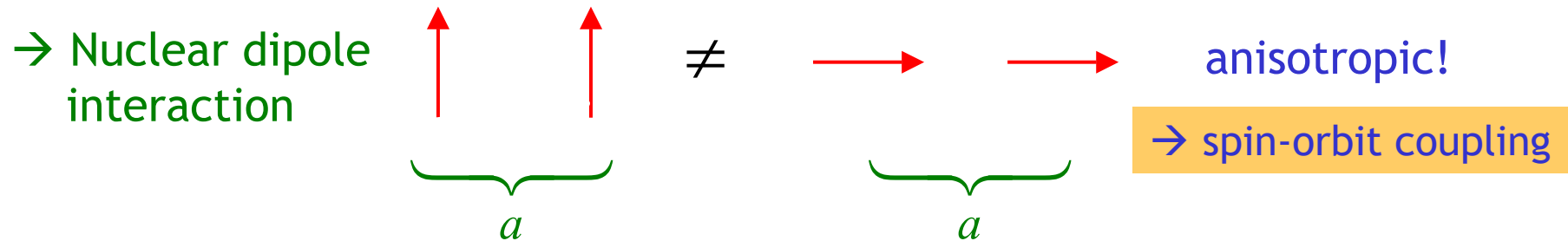
Resolution of the NMR puzzle:  
Macroscopic quantum amplification

# Superfluid $^3\text{He}$ as quantum amplifier

Cooper pairs in  $^3\text{He-A}$



Coupling of  $\hat{d}$ ,  $\hat{l}$  ?

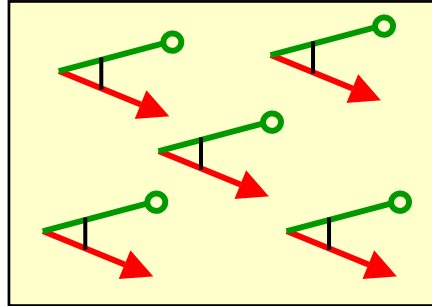


Dipole interaction energy of  $^3\text{He}$  nuclei:  $g_D \sim \frac{\mu^2}{a^3} \approx 10^{-7} \text{ K} \ll T_C$

Unimportant ?!

# Superfluid $^3\text{He}$ as quantum amplifier

Cooper pairs in  $^3\text{He-A}$



- Long-range order in  $\hat{d}, \hat{l}$
- $g_D \sim 10^{-7} K$  lifts degeneracy of relative orientation

Quantum  $\Downarrow$  coherence

$\hat{d}, \hat{l}$  locked in all Cooper pairs

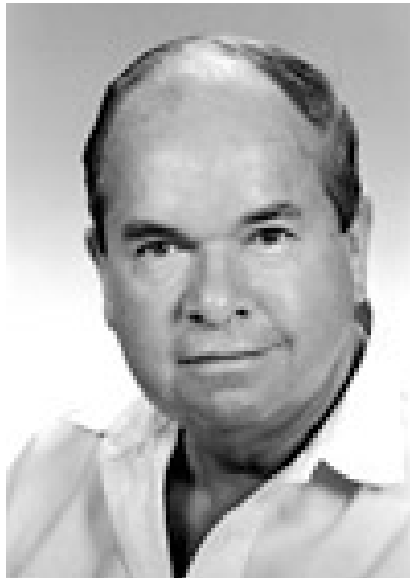


NMR frequency increases:  $\omega^2 = (\gamma H)^2 + g_D \Delta^2(T)$  Leggett (1973)

→ Nuclear dipole interaction macroscopically measurable

## The Nobel Prize in Physics 2003

"for pioneering contributions to the theory of superconductors and superfluids"



Alexei A.  
Abrikosov, USA  
and Russia

b. 1928



Vitaly L.  
Ginzburg,  
Russia

b. 1916



Anthony J.  
Leggett, UK  
and USA

b. 1938



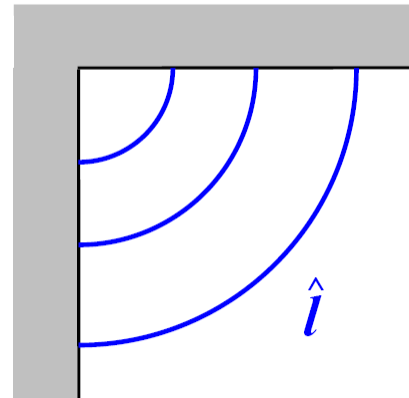
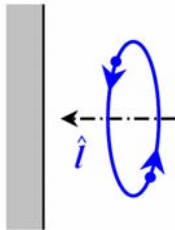
# Order parameter textures and topological defects

## Order parameter textures

Orientation of anisotropy directions  $\hat{d}, \hat{l}$  in  $^3\text{He-A}$ :

Magnetic field  $\rightarrow \hat{d}$

Walls  $\rightarrow \hat{l}$

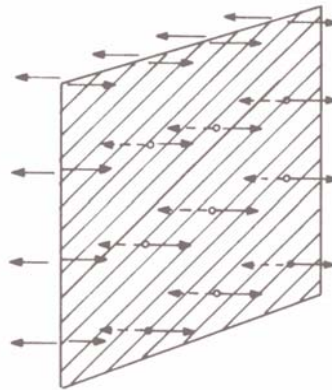


“Textures” in  $\hat{d}, \hat{l}$   $\leftrightarrow$  liquid crystals

$\rightarrow$  Topologically stable defects

# Order parameter textures and topological defects

D=2: domain walls in  $\hat{d}$  or  $\hat{l}$



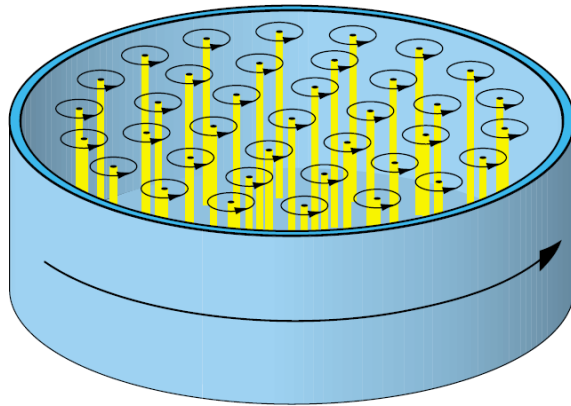
Single domain wall



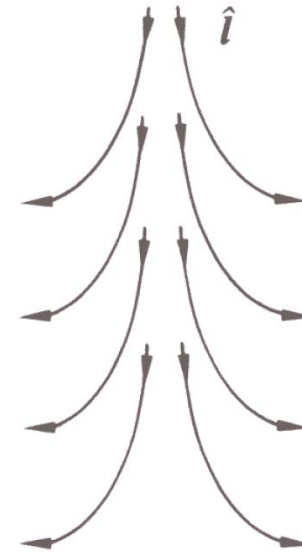
Domain wall lattice

# Order parameter textures and topological defects

D=1: Vortices



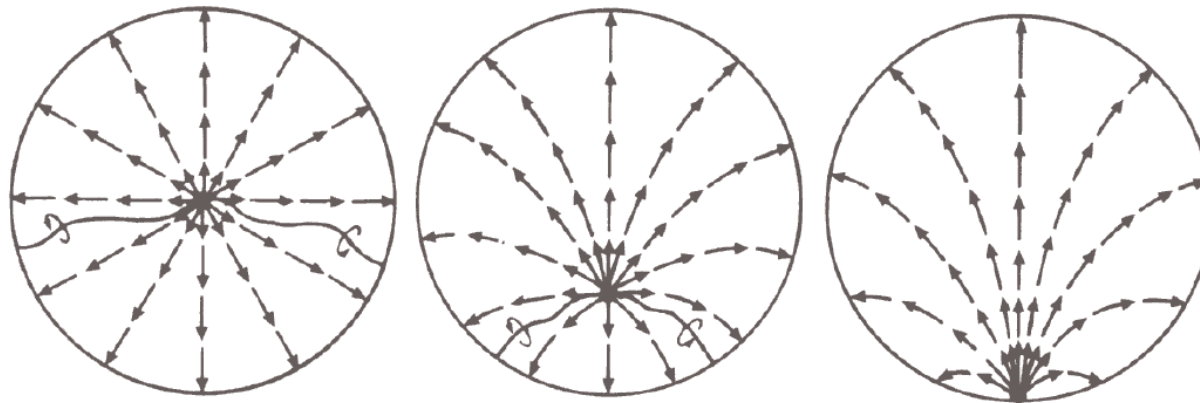
Vortex formation in rotation experiments



e.g., Mermin-Ho vortex  
(non-singular)

# Order parameter textures and topological defects

D=0: Monopoles



e.g., “boojum” in  $\hat{l}$ -texture of  $^3\text{He-A}$

Defect formation by, e.g.,

- geometric constraints
- rotation
- rapid crossing through phase transition

# Universality in continuous phase transitions



High symmetry,  
short-range order

$T > T_c$

Spins:  
para-  
magnetic

Helium:  
normal  
liquid

Universe:  
Unified forces  
and fields

$T = T_c$

Phase transition

Broken symmetry,  
long-range order

ferromagnetic

superfluid

elementary  
particles,  
fundamental  
interactions

Defects: domain  
walls

vortices,  
etc.

cosmic strings,  
etc. Kibble (1976)

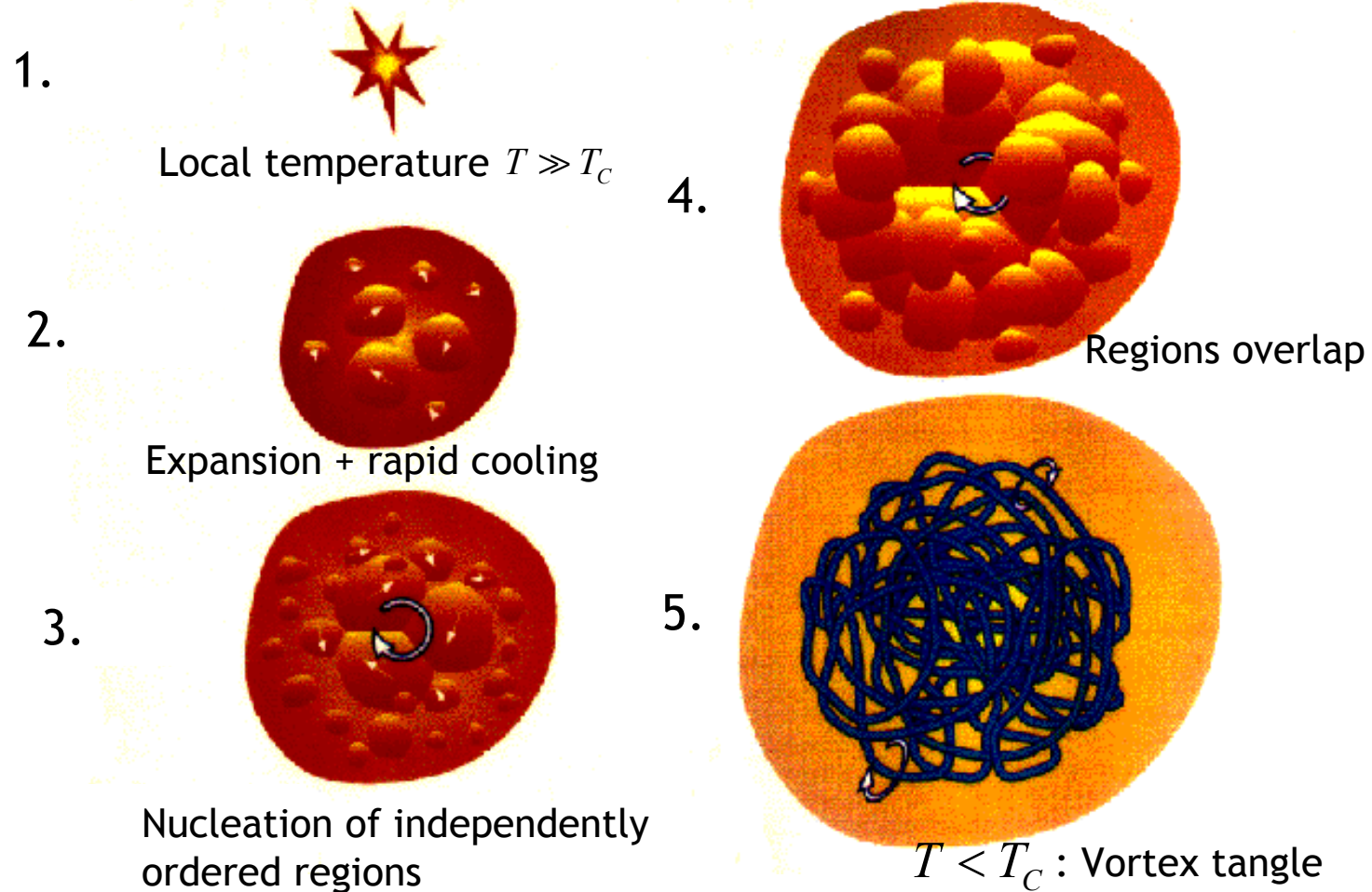
$T < T_c$

nucleation of galaxies?



# Rapid thermal quench through 2. order phase transition

Kibble (1976)



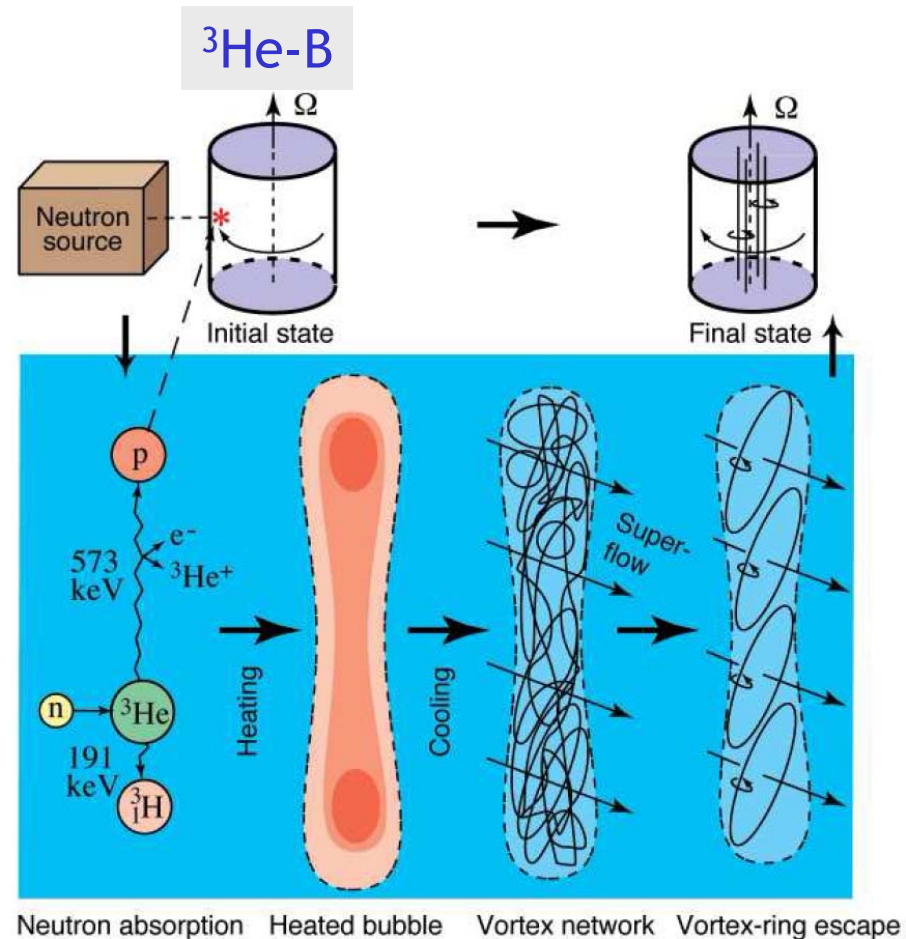
Density of defects

Zurek (1985)

→ "Kibble-Zurek mechanism": How to test?

# Big Bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)



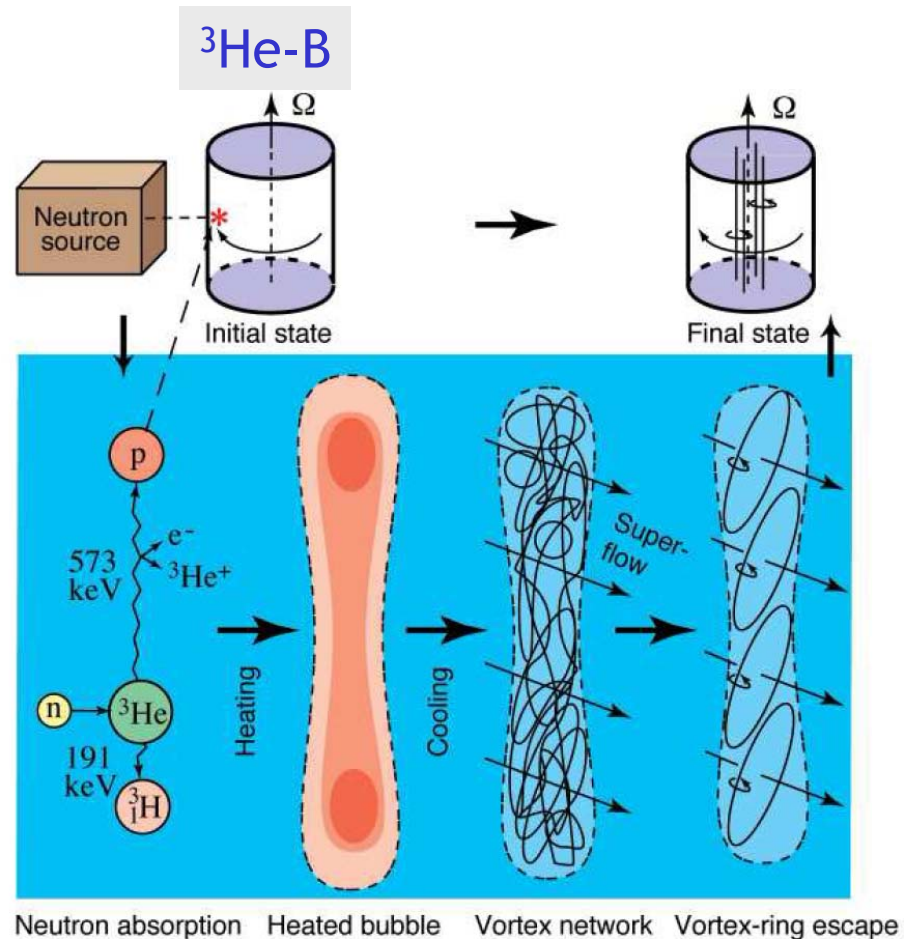
Measured vortex tangle density:

Quantitative support for Kibble-Zurek mechanism



# Big Bang simulation in the low temperature laboratory

Grenoble: Bäuerle *et al.* (1996), Helsinki: Ruutu *et al.* (1996)



Cosmology in the Laboratory (COSLAB)



2001-2006

# Present research on superfluid $^3\text{He}$ :

## Quantum Turbulence

= Turbulence in the absence of viscous dissipation

Test system:  $^3\text{He-B}$

Vinen, Donnelly: *Physics Today* (April, 2007)

# Conclusion

## Superfluid Helium-3:

- Anisotropic superfluid
  - 3 different bulk phases
  - Cooper pairs with internal structure
- Large symmetry group broken
  - Close connections to particle theory
  - Zoo of topological defects
  - Kibble-Zurek mechanism quantitatively verified

