Physica 107B (1981) 659-660
North-Holland Publishing Company

QUASICLASSICAL GREEN'S FUNCTION IN THE BCS PAIRING THEORY

Ulrich Eckern[+] and Albert Schmid

Institut für Theorie der Kondensierten Materie,
Universität Karlsruhe, D-7500 Karlsruhe, BRD

We develop a generalization of the quasiclassical Green's function which allows us to include the transfer of even small amounts of momentum to the particles. We explain the general principle and add a few illustrations.

The pairing theory of Bardeen, Cooper, and Schrieffer has been found extremely successful for superconductors as well as for the low temperature phases of ^{3}He. In general terms, this theory owes its success in quantitative predictions to the fact that nature has made the pairing energy so much smaller than the Fermi energy. Yet, another reason for its success results from the ease - comparatively - by which calculations can be performed. Indeed, most calculations can be simplified by means of approximations which exploit the fact that the Fermi-momentum is extremely large in comparison with the momenta of, say, external perturbations; in other words, it is possible to simplify calculations since the Fermi wave length is negligibly small compared with any other lengths.

A theory which incorporates systematically such approximations from the beginning can be developed on the basis of the Green's function $G(\vec{p},E)$ as introduced by Gorkov. The fact that the Fermi-momentum is extremely large can be understood in the sense that the length of the momentum $\vec{p}$ in the Green's function is of no importance. Consequently, one may argue that it is possible to integrate $G(\vec{p},E)$ with respect to $|\vec{p}|$ with no loss of information. Thus, we obtain the standard quasiclassical Green's function which depends only on the direction of $\vec{p}$.

The quasiclassical theory in this form, however, seems to neglect a transfer of momentum to the particles entirely. Such a neglection has most severe consequences in cases where the transfer of momentum occurs in a systematic way. An example for a systematic transfer is the motion of a charged particle in a magnetic field which, no matter how small, leads to closed orbits. Another example is provided by the transition to a moving frame of reference where each particle acquires an additional momentum which leads to essentially different trajectories.

Thus, we have developed a generalized quasiclassical theory where the Fermi energy, that is to say, the chemical potential μ is considered as an irrelevant variable. Consequently, we define the generalized quasiclassical Green's function as follows

$$g(\vec{p},E) = \frac{i}{\pi}\int d\mu\, G(\vec{p},E) \quad . \tag{1}$$

Let us now recall that the Green's function generally depend on the center-of-mass coordinates $(\vec{r},t)$ and that formally, $(\vec{p},E)$ are the Fourier conjugate variables of the coordinate difference. Therefore, quantities which are of the same formal structure as the Green's function are multiplicatively connected by means of a star-product which is explained as follows

$$A*B = A\cdot B - \frac{i\hbar}{2}\left[\left(\frac{\partial A}{\partial \vec{p}}\frac{\partial B}{\partial \vec{r}} - \frac{\partial A}{\partial E}\frac{\partial B}{\partial t}\right) - \left(\frac{\partial A}{\partial \vec{r}}\frac{\partial B}{\partial \vec{p}} - \frac{\partial A}{\partial t}\frac{\partial B}{\partial E}\right)\right] + \ldots \tag{2}$$

The quasiclassical Green's function is normalized in the sense that

$$g * g = 1 \quad . \tag{3}$$

Furthermore, it obeys an equation of motion in the form

$$\left[E - \frac{p^2}{2m} - \Sigma \overset{*}{,} g\right]_- = 0 \quad , \tag{4}$$

where Σ is the self-energy. Eqs. (3)and(4) determine g completely.

It is known from Gorkov's theory, that the Green's function is properly speaking, a matrix in particle-hole space and possibly, in spin space too. This property requires minor modifications, for instance, in Eq. (4). Less known is the fact that the discussion of time-dependent phenomena is greatly facilitated if one introduces in addition to the modifications mentioned above a Green's function which is a matrix in, let us say, Keldysh's space, namely

$$\bar{g} = \begin{pmatrix} g^R & g^K \\ 0 & g^A \end{pmatrix} \tag{5}$$

where g^R and g^A are the retarded and advanced Green's function and where g^K has been introduced by Keldysh[1)].

It follows from Eq. (3), that g^K can be written as

$$g^K = g^R * h - h * g^A. \qquad (6)$$

Further inspection shows that f = (1-h)/2 has the meaning of a quasiparticle distribution function. These relations allows us to construct a Boltzmann equation in a rather straight forward way from Eq. (4).

The generalized quasiclassical theory possesses the same symmetries as the original problem. This has been demonstrated explicitly for Galilei transformations, rotations, and gauge transformations.

We have also studied the quasiparticle flow pattern during the motion of the orbital vector in an anisotropic pairing state which is commonly known as the ABM state. We also learn in what meaning one may assign an intrinsic angular momentum to the Cooper pairs.

We have also derived a Boltzmann equation for a superconductor which should allow us to calculate the Hall effect on moving vortices.

REFERENCES:

+ Present Address: Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, N.Y. 14853.

1) L.V. Keldysh, Sov. Phys. JETP 20, 1018 (1965).